TEACHERS' MANAGEMENT OF THE MEANING CONSTRUCTION IN THE MATHEMATICS CLASSROOM

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The five studies presented here focus on aspects of the ways teachers manage the construction of meaning in the mathematics classroom: the ways they handle the epistemological features of mathematics and deal with pupils' work and errors and the communicative patterns they adopt. The results show that the management of the content of the subject matter often distorts the mathematical meaning and it is dialectically related to the communicative practices employed.

Theoretical Issues

Research in mathematics education is increasingly focusing on the learning and teaching processes as they are interactively constituted in the classroom, recognizing that the latter is essential to the study of the construction of school mathematical knowledge. In this perspective, the classroom is usually seen as a social context in which mathematical knowledge is negotiated and constructed, while, at the same time, students and teachers are constructed and positioned with respect to that knowledge.

It is widely accepted today that cognitive gains are made in socio-cultural contexts in which teachers enable learning by drawing learners forward in their own ways with appropriate activities. In this respect, in order to become aware of the dynamics of the mathematics classroom and in particular of how students develop their own autonomous mathematical identity, a sociological perspective on mathematical activity is especially valuable. Various researchers have attempted to investigate the activity in the mathematics classroom from such a perspective. For example, Sullivan (1999) focused on the classroom tasks, arguing that in the mathematics classroom, even though the focus may be on particular concepts, the enacting of these concepts requires attention being paid to generalizing, problem solving, linking and synthesizing as well as to a variety of situations in which the concepts are applicable.

Sullivan and Mousley (2001), adapting a framework suggested by Yackel and Cobb (1996), identified two complementary norms of activity in the mathematics classrooms. The first, named 'mathematical norms', concerns "the principles, generalizations, processes and products which form the basis of the mathematics curriculum". The second, called 'socio-cultural norms', refers to the "usual practices, organizational routines and modes of communication that impact on the approaches to learning teachers choose, the types of responses they value, their views about legitimacy of knowledge produced, the responsibility of individual learners and their acceptance of risk-taking and errors". One aspect of these latter norms is the constraints imposed by students on the mathematics teacher, which shape his/her thinking with respect to planning and teaching. For example, teachers tend to avoid negative student reactions (Shroyer, 1982) and to adjust their teaching to the preferred learning style of the students. In relation to the latter, Doyle (1986) reported that working on a task pupils try to reduce their risk of failure by seeking to increase the explicitness of the task requirements as well as the level of accountability, thereby narrowing the demands of the task. Reacting to this, teachers tend to select tasks that are familiar and easy to the pupils.

In the studies reported below, an attempt is made to examine aspects of the mathematical on the one hand and of the socio-cultural classroom norms on the other.

The Studies

All five studies presented here utilized the same set of data. The first two focus on the mathematical norms and in particular on the ways teachers tend to handle fundamental epistemological features of the subject matter both at the primary and the secondary level in two rather different mathematical contexts, those of algebra and geometry. In relation to the socio-cultural norms, the same data were analyzed in terms of the ways teachers chose to intervene during pupils' classroom work and also to treat pupils' errors (studies 3 & 4 respectively). Finally, seeking to understand how the two types of norms come together in the classroom, we looked at the interactions between teachers and pupils, in order to examine how this interaction shapes the epistemological features of mathematics as well as the intervention practices adopted by teachers (study 5).

The data for the studies came from a large project focusing on the mathematics teaching in the nine years of the Greek compulsory educational system (6 – 15 year olds) and aiming at investigating the possibility of applying alternative, pupil-centered mathematics teaching approaches in the Greek school. The data collected consisted of 48 mathematics lessons (28 primary and 20 secondary) given by 23 teachers (11 primary and 12 secondary), observed in various classes of the last two grades of primary school and of all three grades of high school for over a month in the northern part of Greece. For each teacher, at least two 45 minutes sessions on different topics were observed; these were then videotaped and transcribed.

In the following, for each of the five studies, some theoretical issues, the research question(s), the analysis of the data and some basic conclusions are briefly presented. Finally, an attempt to draw some general conclusions with respect to the mathematical and the socio-cultural norms of the mathematics classroom is made.

Studies 1 & 2: the management of the epistemological features of mathematics in two contexts

Pupils learn what is important in mathematics via interpreting the classroom events, attributing to each of them a value proportional to its usefulness to the mathematics lesson (Sierprinska & Lerman, 1996). These interpretations concern the meaning of concepts and processes as well as their nature and value. In this respect, the study of the nature and the organisation of the mathematical content become of particular importance. Such a study requires an analysis of the ways in which the epistemological elements of mathematics, that is, the nature, meaning and definitions of the mathematical concepts, the theorems and the solving, proving and validation procedures emerge in the mathematics classroom.

In study 1, the analysis carried out focused on two dimensions: (a) the organisation and interrelationships of the various elements of the mathematical content (concepts, definitions of concepts and theorems) and (b) the organisation and selection of the elements of the mathematical activities (solving, proving and validation processes). The results showed that both in primary and secondary classes, the way in which the epistemological features of mathematics are presented and dealt with does not allow them to be distinguished from one another with respect to their meaning and function in mathematics. Furthermore, the activity developed in the mathematics classroom is often, almost entirely, deprived of the characteristics of mathematical processes, which have to do with the pursuit of solving and proving processes, as well as checking and confirming.

The following episodes substantiate the preceding points. In episode 1.1, the concept is reduced to a counting process, in episode 1.2, the definition and the property are placed on the same level through questioning and in episode 1.3, measuring is suggested as a proving process.

Episode 1.1. The teacher presents the area of a rectangle as a process of counting and through this he generalises (11 years old):

T(eacher). *…Count the (number of) boxes (square centimetres), how many boxes are there?*

P(upil). *There are 12 boxes.*

T. *So, the area of the rectangle is 12 square centimetres. Then, how can we find the area of a rectangle? What do we have to multiply?*

P. ....

T. *The area of the rectangle is: base by height and we measure it in square centimetres or square metres.*

Episode 1.2. The way the teacher elicits the definition and the properties is absolutely muddled (12 years old).

T. *... thus, what do we have in the isosceles triangle?*

P. *Two sides equal?*

T. *Yes, and what else?*

P. *And two angles equal*

T. (.. a little later) *So, what do we call equilateral triangle?*

P. *The one that has three equal angles and three equal sides.*

Episode 1.3. Pupils are asked to categorise various triangles according to the size of their angles, which they first measure (11 years old).

T. *Now, take the protractor and measure the angles and decided what kind of triangle it is.*

P. *In the first triangle there is a* 90^0 *angle.*

T. *How did you find it?*

P. *It looks like it, it seems to be a right angle.*

T. *In mathematics, we can't claim something without being able to prove it. Take the protractor please, measure the angle and tell me whether it is in fact* 90^0.

The results of study 1 give rise to an interesting question: how is this homogeneity realised in the context of the two main branches of the school mathematics curriculum, that is, of algebra and geometry, given that these two areas differ epistemologically and promote rather distinct patterns of thinking? School algebra is usually introduced as generalised arithmetic and is often described as "an epistemological transition from a procedural to a relational perspective" (Arzarello, 1998). School geometry, on the other hand, places great emphasis on the visual aspects of the subject matter, thus leading children to dealing with the geometrical objects in a visual-perceptive rather than relational – analytical way (e.g. Hershkowits et al, 1996).

Study 2 attempted to investigate the way in which the epistemological features of mathematics are treated in the context of school algebra and geometry. The corresponding analysis of the data showed that definitions and theorems are often reduced to processes of manipulation in algebra and of visual recognition or of drawing in geometry. Furthermore, the way the mathematical knowledge is handled in the classroom appears to foster reconnoitring and morphological elements in algebra and handling/manipulative elements in geometry. These suggest that the management of the mathematical knowledge in the two contexts does not only prevent the differentiation of their epistemological elements, but, on the contrary, it unifies them.

The following episodes are indicative of the ways in which the teachers of the sample dealt with the mathematical content in the two contexts. In the first episode, the mathematical knowledge is presented as a set of ready-made instructions of factual character, the emphasis being placed on the morphological elements of the transformations of the algebraic expressions. In the second episode, the definition is reduced to a manipulative step-by-step instruction on the actual drawing of the altitude of a triangle.

Episode 2.1.(algebra). The teacher starts by defining factorisation (14 years old).

T. *The factorisation of an algebraic expression consists of its transformation to a product of two or more other algebraic expressions.*

A little later the mathematical method turns into a process:

T. *We will study about ten cases, will see them one-by-one and we will learn practical rules... so, if we are given this, we will do that, and so on.*

In the end of the lesson, the two cases studied become rules:

T. *Let me make one or two observations: we notice that when the powers of the same letter appear in all the terms of the polynomial, then the power of this letter with the smallest index comes out of the bracket. The second case concerned the grouping of the terms ….The common factors of each group come out of the bracket and what remains inside the bracket in each group is the same.*

Episode 2.2. (*geometry).* The teacher starts by giving the definition of an altitude of a triangle. However, the definition is virtually "destroyed" in the following, as the focus of the lesson moves on the way in which the altitude is to be drawn (13 years old).

T. *We first give the definition. What is the height of a triangle: we call height of a triangle the distance of a vertex from its opposite side.*

A child repeats. The teacher works on the drawn shape.

T. *So, how are we going to place the ruler, here, watch ...The one side will go through the point (vertex) and the other (should be placed) on the side. We will do it practically, take the (right-angled) ruler.*

Studies 3 & 4: teachers' practices in dealing with pupils' work and errors - interventions

In trying to understand the complexity of the mathematics classroom, many studies carry out an analysis of teaching episodes, focusing in particular on the way teachers intervene in order to support or guide pupils' work. This analysis has identified a number of facts, which appear to be very common in the mathematics classroom and which have certain consequences for the mathematics generated, the pupils' attitude towards mathematics and their knowledge about the mathematical knowledge. The relevant literature shows that all these types of interventions function as external indicators often misinterpreted by the pupils, who tend to adapt them to their existing system of knowledge, this requiring less effort. As a consequence, the mathematical content of the task is often simplified and possibly distorted and its cognitive value is reduced (e.g., Diezman et al, 2001). The above suggests that it is very important for the mathematical knowledge elaborated within the classroom and for the teachers' teaching practices to systematically identify “types” of critical teaching phases and their management by the teachers. To this purpose, we looked at teachers’ interventions in two different and at the same time significant occasions of the classroom activity.

(a) When difficulty emerges during the pupils’ engagement with a task or the course of development does not follow the path intended by the teacher: These are the moments that teachers tend to mostly intervene in a number of ways, e.g., by offering premature or of local character explanations (e.g., Margolinas, 1999), addressing the competent students to secure the development of the lesson according to the initial plan, etc. In study 3, we attempted to classify teachers’ interventions according to the ‘degree of freedom’ they provided. The analysis of the data showed that the dominant types of interventions are: focus on techniques, processes and representations; step-by step guidance; demonstration of the problem’s solution. Furthermore, it is important to note that in general, the type of intervention made by the teacher is related to the students’ attitude and actions with respect to the situation or problem at hand. However, the examination of the teaching episodes revealed that the teacher often intervened for no apparent reason concerning the pupils having difficulty or being stuck. In other words, s/he interfered independently of their action.

The two episodes below offer an insight into the ways in which teachers intervene in pupils’ work. In both of them, the teacher directs tightly pupils’ thinking, allowing very little space for them to formulate ideas and complete their reasoning. Furthermore, in the first one, he prevents the justification of the results on the basis of the properties of a triangle, whereas in the second she slips into a process of ending up to the required result.

Episode 3.1. The activity requires the calculation of the size of the angles of a right-angled and isosceles triangle (only the right angle is marked) (12 years old).

T. *Pay attention. This triangle has two characteristics. First, what type of triangle is this Nic, with respect to its angles?*

P. *Right-angled*

T. *Right-angled. With respect to its sides, what type of triangle is this? Tania?*

P. *Isosceles*

T. *Isosceles. Well done Tania. That is, this triangle is right-angled and isosceles. And we know one of the angles, the right angle, Michael?*

P. *Angles b and c…*

T. *Yes…*

P. *They are each* 45^o

T. *But why?*

P. *Ehhh.. Because …*

T. *The triangle is …*

P. *The triangle is right-angled and isosceles.*

Episode 3.2. Pupils are invited to conclude that the sum of the angles of a triangle is 180^o. They initially estimate the sum of the three angles of special cases of triangles and then measure to confirm their estimations (11 years old).

T. *George, what did you find for triangle B. Tell us.*

P. 90^o

T. *Why do you say* 90^o*? All together, eh?*

P. *All together?*

T. *Doesn’t the exercise ask you for the sum of the angles?*

P. *Because they are small.*

T. *The angles are small. Right. Tell us Chris.*

P. *130^o madam.*

T. *About. Why my boy?*

P. *Madam, estimated by the eye.*

T. *By the eye, right. Charoula?*

P. *180^o, madam*

T. *Why 180^o Charoula?*

P. *Because, for the right angle, I say 90^o, for the other one which is acute, because it is very small, I say 10^o and for the other one towards the right angle … but it is acute, I say 80^o.*

T. *About this, I don't know, it might be correct too. ….* Other pupils go on like this suggesting 60^o, 120^o, 185^o, 150^o, but the teacher doesn't ask for any more explanations, she simply says:

P. *Did you estimate it by the eye?*

(b) In dealing with pupils' errors and in using validation procedures in the mathematics classroom: Both of these aspects of mathematics instruction are of great importance for the classroom construction of the mathematical meaning as they are related to the notion of transfer of control from the teacher to the pupils, a decisive factor in the development of autonomy and substantial thinking by the children. This is because it allows classrooms to become less judgmental and shifts responsibility for making sensible contributions to the children.

For the purposes of study 4, the transcripts of the mathematics lessons were analyzed in relation to the treatment of the pupils' errors by the teacher in two phases: before the error was made and after. The analysis of the episodes revealed that teachers keep for themselves the control over errors by warning and directing pupils or making the corrections themselves. That is, they seem to believe that errors are something to be avoided. In this context, they often do not pay attention to pupils' contributions, thus missing opportunities for a fruitful interaction in the construction of mathematical meaning. As a result, teachers invariably use morphological or procedural rather than conceptual elements for the elaboration of mathematical meaning. This practice allows them to keep their leading role intact in the construction of mathematical knowledge.

The following episodes underline this type of teaching practice.

Episode 4.1. The class is trying to simplify an algebraic expression.

T. *In the worksheet I gave you, I have included a case where … there is a minus in front of the expression (describes). What will I do?*

P. *–2a*

T. *-a-1 or, if I don't want to change it immediately -(a+1). You should be very careful. This is where most of the errors appear. Solve as many equations as you can, starting from the simplest ones.*

Episode 4.2. The pupils are trying to find the values of *a* in order for the denominator to be $\neq 0$.

P. $a^2-1 = 0 \Rightarrow a^2=1$

T. *This is one of the ways. Are we sure that we will not lose the root? In the denominator? And what do we say then?*

P. *Will we say a=0?*

T. *Square root of 1. Thus, we get the 1. How will we get the –1?*

P. *We don't get the –1, because –1 times –1 equals plus…*

T. *We are fishing in unclear waters. Any safer way?*

P. *Shall we put plus?*

T. *No. The a^2-1 can be written as a^2-1^2. Does this expression remind you of anything?*

Pupils. *Difference of squares.*

Episode 4.3. The pupils work with a problem of factorisation.

P. *Madam, in x^2-2x, if we write x times x equals 2x? The x is cancelled and then x=2.*

T. *Be careful! Which x' s are going?… priority of operations… first we multiply.*

P. Madam, we will do x^2=2x … x.x=2x

T. *But you have a root! It is not allowed! All right? You lose a root. Don't do this kind of cancellation, because you lose roots. All right? However, when we take out the common factor, we don't lose the root.*

Study 5: linking the management of the epistemological features with the communicative patterns in the mathematics classroom

The previous studies suggested that there is a relationship between the mathematical content and the social structure of the classroom as determined by the norms of interaction and communication employed. In this final study, an attempt was made to examine the interaction in the mathematics classroom. To this purpose, the transcripts of the lessons were analysed by looking at the interplay between the communicative patterns and the management of the mathematical knowledge by the teacher in two phases of the pupils' engagement with the activities and with respect to the epistemological differentiation achieved: (a) at the completion of a certain activity in a unit of activities and (b) at the completion of a whole unit and the generalisation of the results. The analysis of the data indicated that there is a dialectic relation between the communicative pattern and the management of the mathematical content within the classroom.

Most of the episodes reported in the previous studies (e.g., episodes 3.1 and 4.3) support this as they disclose that the type of interaction which dominates the mathematics classroom is not negotiable and it shapes and at the same time is shaped by the way teachers organise and manage the mathematical knowledge targeted.

Concluding Remarks

A number of studies have put emphasis on the importance of the interactive patterns of teaching and learning in the acquisition and the development of mathematical knowledge. However, as many researchers argue, there should not be a total shift of analytical attention from subject matter-structure to social-interactional structure. This is because there is then «a risk of destroying theoretical mathematical meaning by a reduction and a hypostasis of mathematical relations instead of inducing an enrichment of meaning by the interactive construction of new and more general relations» (Steinbring, 1998). Taking this a little further, the results of our studies suggest that there is interplay between the epistemological organisation of the mathematical content and the organisation of the mathematics classroom. More specifically, the lack of differentiation among the elements of the mathematical content and its mixture with morphological, procedural and management elements of the classroom mathematical activity renders to the latter a dominant status in the activity. Teachers' interventions when students face difficulties as well as their management of the validation procedures and of the errors made by the pupils tend to focus predominately on these elements (seen as more effective indicators of production of mathematical knowledge), thus reinforcing them and consequently distorting the mathematical meaning. Hence, by dominating the mathematical activity, these elements become necessary to the students, that is, they acquire the status of 'terms' for the everyday organisation of the mathematics classroom with respect to what is negotiated each time. In other words, they become a feature of the didactic contract, which constitutes "a system of reciprocal obligation… specific to the 'content', the target mathematical knowledge" Brousseau (1997).

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