

The Dilemma of Under-prepared Teachers of Elementary Mathematics

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Many factors contribute to a general phobia of learning and teaching of mathematics by pre-service teachers. This study sought to elucidate the essential components needed for success in a course for elementary teachers of mathematics. While results indicated that the factors of cognitive ability as determined by Guilford's Structure of the Intellect (SOI) assessment and self-regulation skills as determined by the Learning and Study Strategies Inventory (LASSI) were not statistically robust, significance was determined through correlation analysis and regression. The data seem to suggest, that coupled with high motivation and self-discipline, pre-service teachers that have a solid foundation in algebraic fundamentals experience a higher degree of success in a course for the learning and teaching of elementary mathematics than their counterparts that are not so prepared.

California has been in crisis mode for the last few years, especially in mathematics education. While budget cuts have created additional chaos, mandatory class downsizing has produced an accelerated need for qualified teachers, especially in elementary mathematics. Of particular concern is the number of "provisional" teachers that have been and will be hired. In sum, the issue in question is one of competence in the mathematics discipline. Certainly methods classes are essential, but a proper foundation in the mathematics discipline is indispensable. In the United States, this need becomes even more acute with the renewed emphasis on formally teaching algebra in the 7th and 8th grades (Carre' & Ernest, 1993; NCTM, 2000).

Many students in the liberal studies program consider math-related courses as the most dreadful and most feared courses in the curriculum. Bibby (1986) concurs when he cites a typical student's reaction toward mathematics as 'it reminded her of a boa constrictor which slobbers its victim before swallowing them' (p.60). However, in spite of the fear and foreboding, the subjects covered in these courses are crucial to academic and professional progress toward a teaching credential.

The California content standards for teaching mathematics in the elementary school are explicit in what knowledge base must be mastered. Moreover, one cannot teach what one does not know. Research indicates that test anxiety, poor study skills, lack of motivation or concentration, and a general phobia toward mathematics are all factors that contribute to the lack of mastery of math content (Arvidson, 1997; Tobias, 1978).

Parker Palmer (1988) has astutely noted that there exists a symbiotic relationship between the content, the instructor and the learner. While the skirmishes around content standards and under-prepared students are well documented (TIMSS, 1996), the preparation of the pre-service teacher is the critical element for research in this study.

According to Meeker (1979), the research clearly indicates that most learning failures occur because the learner is not prepared to learn. The same can be applied to pre-service teachers that are preparing to teach math at the elementary school level. Why do some pre-service teachers succeed in demonstrating competence in K-8 mathematics while others struggle to understand and subsequently fail to competently explain the math concepts they are required to teach?

Fundamentally, the purpose of this study was to investigate the correlations between competence in elementary mathematics and numerical ability, logic and form reasoning, cognition of symbolic systems, evaluation of symbolic systems, convergent production of symbolic systems, information processing, selecting main ideas, test strategies, attitude, motivation, anxiety, concentration, time management, self-testing, study aids, previous success in algebra and attitudes toward mathematics. Considering the myriad of factors that are involved in learning mathematics, a second purpose was to find any identifiable characteristics that contribute to high achievement in the course ‘mathematics for elementary teachers’.

Method

Procedure

At the beginning of the semester, students in the Mathematics for Elementary Teachers course were asked to participate in a study investigating cognitive abilities and study skills through a series of assessment instruments. At this point students completed a survey that included information on sex, age, career goal, highest math class completed, attitude towards mathematics and the grade received in their last algebra class. The students were informed regarding the purpose of the study and were asked to identify themselves by their student ID number. It should be noted that research indicates that students with poor cognitive abilities frequently overestimate their performance. This could diminish the reliability of self-reporting regarding their aptitude in mathematics (Kruger & Dunning, 1999).

Using the Structure of the Intellect theory (Guilford, 1967) as the cognitive model, the SOI learning abilities test (Meeker, 1975) was used to test each group’s specific cognitive abilities. In addition, the LASSI assessment instrument was also given to assess behaviors, attitudes and beliefs that relate to successful learning (Weinstein, 1987).

Subsequent to both of these assessments, subjects were given three examinations during the course of the semester to determine their mastery of content required for K-8 teachers of mathematics. In addition, a course grade that also included teaching a math lesson, a written review of the state framework for mathematics, completed homework from a textbook and a monthly mathematics activity packet was used to measure the subjects performance in the class. The course grade was included in the data set at the end of the semester after the final exam was graded.

Subjects

Subjects were undergraduates enrolled in a liberal studies course, Mathematics for Elementary Teachers at Azusa Pacific University in Southern California. Further, 81 students were enrolled in three sections of an undergraduate course covering elementary school mathematics for pre-service teachers. As previously noted, their performance was measured on three major exams as well as a course grade that included teaching a math lesson, a written review of the state framework for mathematics, completed homework from a textbook and a monthly mathematics activity packet.

Data from students who completed both assessment instruments and all three examinations were retained for analysis. Sixty-eight of the 81 students enrolled met these inclusion criteria. Although students who met the inclusion criteria may have differed from

the other students in the class, multivariate analysis of variance revealed no significant differences on examination grades or final grades between students who met and students who failed to meet the inclusion criteria.

Instruments

The Learning and Study Strategies Inventory (LASSI) is a 10-scale, 80 item assessment of students' awareness about and use of learning and study strategies related to skill, will and self-regulation components of strategic learning. Moreover, these skills include behaviors, attitudes, and beliefs that relate to successful learning (Weinstein, 1987).

Specifically, the LASSI scales related to the skill component of strategic learning are: information processing, selecting main ideas, and test strategies. These scales examine learning strategies, skills and thought processes related to identifying, acquiring and constructing meaning for important new information, ideas and procedures, and how they prepare for and demonstrate their new knowledge on tests of other evaluative procedures.

The LASSI scales related to the will component of strategic learning are: attitude, motivation and anxiety. These scales measure students' receptivity to learning new information, their attitudes and interest in college, their diligence, self-discipline, and willingness to exert the effort necessary to successfully complete academic requirements, and the degree to which they worry about their academic performance.

The LASSI scales related to the self-regulation component of strategic learning are: concentration; time management; self-testing and study aids. These scales measure how students manage, or self-regulate and control, the whole learning process through using their time effectively, focusing their attention and maintaining their concentration over time, checking to see if they have met the learning demands for a class, an assignment or a test, and using study supports such as review sessions, tutors or special features of a textbook.

Each scale, with the exception of the Selecting Main Ideas Scale, has 8 items. Selecting Main Ideas has 5 items. Coefficient Alphas for the scales range from a low of 0.68 to high of 0.86 and test-retest correlation coefficients for the scales range from a low of 0.72 to a high of 0.85, demonstrating a high degree of stability for the scale scores.

The second assessment instrument that was used in the study is based on Guilford's (1971) Structure of Intellect theory. J.P. Guilford first developed the theory in the United States in the early 1940's. In its first documented application, the Structure of Intellect assessment method was able to reduce the failure rate of US Air Corps recruits from 33 in 100 to 3 in 100. This was achieved by streamlining the selection criteria to better reflect the mental abilities needed to be a pilot (Meeker & Meeker, 1999).

Subsequently, Guilford & Hoepfner (1971) developed a wide variety of psychometric tests to measure the specific abilities predicted by the theory. The theory defines intelligence as a juxtaposition of operations, products and contents for processing different kinds of information in various ways. Further, Guilford's structure of intellect model has been applied in programs to enhance thinking skills such as the SOI (structure of intellect), designed by Meeker (1969, 1999). The SOI is a standardized assessment instrument. The primary purpose of the test is to provide an accurate profile of the examinee's cognitive learning abilities.

Specifically, the SOI encompasses 26 separate tests in 5 different areas. However, this researcher chose to focus on two areas, that of arithmetic and mathematics. In the area of

arithmetic, three test were utilized, that of Numeric Facts (CSS), Numeric Judgment (ESS), and Numeric Application (NSS). In the area of mathematics, the Form Reasoning (NSI) test was used.

Specifically, CSS is the cognition of symbolic systems. This is a test of comprehension of numerical progressions. In this subtest, the student must find the rule that is generating a number series. Arithmetic ability is required, but only elementary rote skills. This subset provides information on how well students have mastered rote skills in arithmetic.

ESS is the evaluation of symbolic systems. In this subtest the student is evaluating systems of numbers. The ability to select the correct principle is being tested. Rules are presented and the student examines series of numbers to find the series that has been described by the rule. The task requires math skills. This ability relates to students' problem-solving skills.

NSS is the convergent production of symbolic systems. This is a test of the ability to solve complicated arithmetic problems that do not depend on verbal skills. The student is presented with a starting number and a target number to be obtained through a sequence of numerical operations. The task requires skill with signed numbers and selection of correct principles for solutions. Students who have difficulty on this subtest may have problems in seriation or conservation.

NSI is the convergent production of symbolic implications. This is a test of the ability to deduce the solution to a symbolic problem. This subtest involves logic and form reasoning. It requires the student to perform a substitution of a given equivalence or equivalencies to arrive at the correct answer. In the first column, however, substitution is not required; the student simply looks at the top of the column for the correct answer. This subtest predicts the ability to work with commutation and can be used as a screening test for placement into algebra if CSS, ESS and NSS computation skills are good.

Moreover, since both males and females were involved in the study, Maxwell (1984) confirmed in her research that there were no sex differences in true variance, error/uniqueness variance, or in factor inter-correlations. In sum, this finding suggests that the SOI-LA test is the same for both females and males.

Results

Statistically significant positive correlations between course grade and independent variables were obtained for SOI-ESS ($r= 0.536$, $p< 0.001$), SOI-NSS ($r= 0.353$, $p< 0.01$), SOI-NSI ($r= 0.372$, $p< 0.01$), LASSI-MOT ($r= 0.387$, $p< 0.01$), and last algebra class grade ($r= 0.404$, $p< 0.01$)(see table 1). In addition, paired t tests revealed no significant differences between student's final grade in the course and the student's grades on the exams.

Table 1

Correlations Between Course Grade and Cognitive Variables

Variable	Course Grade	Algebra Grade
SOI-ESS	0.536***	0.483***
SOI-NSS	0.353 **	0.369**
SOI-NSI	0.372**	0.273*
LASSI-MOT	0.387**	0.289*
Algebra Grade	0.404**	1.000

(* $p< 0.05$, ** $p< 0.01$, *** $p< 0.001$)

Overall regression analysis showed that the model with the 5 predictors accounted for 34% of the variance ($R^2 = 0.342$). Moreover, the F value was significant at 6.977 ($p < 0.001$) with 5 degrees of freedom, validating that the model with the 5 predictors is statistically significant. Regression also reported that SOI-ESS was the strongest predictor ($p < 0.05$) and LASSI-MOT was borderline significant ($p = 0.05$) (see table 2). Interestingly, ‘last algebra grade’ was not reported to be significant in the regression analysis even though it was the second highest correlated variable to course grade. In sum, when all 5 predictors were controlled for, regression does not show the predictive power of last algebra class grade.

It is hypothesized that since the variables SOI-ESS, SOI-NSS, SOI-NSI, and LASSI-MOT were not controlled for in the correlations, the last algebra class grade contained some of the predictive power of the other variables. Similar results were found in a study of SI components in ninth-grade mathematics achievement (Guilford, Hoepfner, & Petersen, 1965). For example, the abilities of CSS, CSI, NSS, NST, and NSI were found to be relevant to success in Algebra. In addition, according to Guilford (1967, 1982) CSS, ESS, NSS and ESC are used to assess rote arithmetic skills. If all of these and NSI are high, they serve as a good cluster of skills for success in algebra.

Table 2

Regression of Course Grade on the Independent variables

Variable	Beta	T	sig T
Algebra Grade	0.100	0.797	0.428
LASSI-MOT	0.203	1.865	0.067
SOI-ESS	0.337	2.048	0.044
SOI-NSS	0.000	0.002	0.999
SOI-NSI	0.115	0.983	0.329

R = 0.585

$R^2 = 0.342$

F = 6.977 sig F = < 0.001

N = 67

Discussion

The results of this study suggest that a thorough grounding in algebraic skills and knowledge is the biggest single predictor of success in a mathematics for elementary teachers course. While SOI-ESS was the strongest predictor for success in the mathematics for elementary teachers course, it is also an essential component to success in any course requiring algebraic reasoning. ESS, or the judgment of correctness of numerical facts, is the ability to make correct decisions about which of the concepts are needed and the order in which to use them to solve non-verbal math problems. The application of ESS in a course requiring algebra is most easily seen in order of operation problems or in two-step equations. Other requisite skills in algebra are highly diminished in value if the student does not know whether, or when, to add, multiply, divide, etc., in the correct order. In sum, students that are low in ESS have not learned which math rule or principle to use first in solving problems.

A common view of math education is that its main aim is the acquisition of knowledge through the learning of facts. In any given classroom, most measure the effectiveness of math education by testing students’ knowledge. But do these tests really measure student’s

mathematical abilities and understanding? While the learning of facts might be the goal of certain courses, it is not the seminal purpose of math education. Rather the purpose is to improve the mind by acquiring abilities and skills to do things they could not do previously. As Plutarch has astutely noted, “The mind is a fire to be kindled, not a vessel to be filled.” Books and files store many more facts than people do. In fact, they are excellent “vessels,” but that does not make them smart. Being smart is about doing, not just about knowing (Gardner, 1993). It is not enough to know basic operations in mathematics; the skill is knowing what operation is needed and when to apply it (Arvidson, 1999).

The main benefit of learning and doing mathematics is not the specific content; rather it's the fact that it develops the ability to reason precisely and analytically about formally defined abstract structures (Devlin, 2003; Schoenfeld, 1992). Moreover, many times specific topics in mathematics are not as important as having a high level of mathematical sophistication (NCTM, 1989, 2000). For example, the specificity, rigor and logic found in algebra and geometry provide this sophistication in mathematical understanding and, in turn, form the foundation for subsequent math topics to be fully grasped. As this study seems to indicate, all other measures of mathematical fluency trace their way back to the mastery or failure to master the fundamentals of algebraic reasoning. The National Academy of Sciences seminal report *Adding it up: Helping children learn mathematics* concurs, “The formal study of algebra is both the gateway into advanced mathematics and a stumbling block for many students (Kilpatrick, Swafford, & Findell, 2001, p.419).”

In addition, the study also reported the LASSI-MOT variable as a statistically significant factor in a student's success in the mathematics for elementary teachers course. The variable represents the Motivation scale that assesses students' diligence, self-discipline, and willingness to exert the effort necessary to successfully complete academic requirements. The maxim, “To learn math is to do math” assumes a strong work ethic of doing and persevering so that learning will happen. For example, a low scoring student may choose when the work is difficult to either give up or study only the easy parts. Low scoring students on the Motivation scale also lack responsibility for their academic outcomes and many do not know how to set and use goals to accomplish specific tasks (Weinstein, Schulte & Palmer, 1996; Corno, 1992).

While the connection between course grade and the ability to teach mathematics has not been clearly established, the data seem to suggest that, coupled with high motivation and self-discipline, pre-service teachers that have a solid foundation in algebraic fundamentals experience a higher degree of success in the learning and teaching of elementary mathematics than their counterparts that are not so prepared (McLeod, 1992; Fennema & Franke, 1992).

The quality of instruction is a function of teachers' knowledge and use of mathematical content...It depends critically on teachers who understand mathematics, how students learn, and the classroom practices that support that learning...Teachers need to know the mathematics of the curriculum and where the curriculum is headed. They need to understand the connections among mathematical ideas and how they develop. Teachers also need to be able to unpack mathematical content and make visible to students the ideas behind the concepts and procedures (Kilpatrick, Swafford, & Findell, 2001, p.424- 428).

In sum, the implications arising from under-prepared teachers of elementary mathematics cannot be understated. Although children bring important mathematical knowledge with them to class, most of the mathematics they know is learned in school and depends on those who teach it to them. Therefore, improving students' learning in mathematics depends on the capabilities and knowledge of their classroom teachers.

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