KOREAN PRESERVICE ELEMENTARY TEACHERS' UNDERSTANDING OF FRACTION CONCEPTS

Youngyoul Oh (yyoh@gnue.ac.kr)
Gwangju National University of Education, S. Korea

The study reported here analyzed Korean preservice elementary teachers' understanding of notions and representations about fraction concepts. A total of 115 preservice elementary teachers were selected from Gwangju National University of Education, S. Korea. A questionnaire was used to examine their knowledge about fraction concepts and representation abilities. The findings indicated that preservice elementary teachers' understanding of fraction concepts is closely related to their representation abilities. In addition, they tended to view fractions primarily as part out of the whole, and they had significant difficulty in understanding and representations of fractions as operator. Also, the results showed that many preservice elementary teachers regarded the concept of the unit as the same as the whole.

INTRODUCTION

In the present study the researcher investigated the depths of Korean preservice elementary teachers' understanding about fraction concepts and representation abilities. Teachers' knowledge about mathematical concepts greatly influences on their teaching practice and the quality of students' learning. Likewise, teachers' ability to represent mathematical ideas plays a key role in helping students learn mathematics in a meaningful way. Studies (e.g., NCTM, 2000; Ma, 1999; Ball, 1990) about teaching practice revealed that teachers' profound understanding about mathematical concepts such as fractions is required for effective teaching, and their knowledge of representations is considered as an important factor for improving teaching practice.

Throughout the last decade, the notion of good mathematics teaching has greatly changed from emphasizing skill mastering for correct answers toward focusing on meaningful understanding about mathematical concepts. Thus, the role of mathematics teachers is viewed as learning facilitators changed from knowledge distributor in the traditional perspective. However, it seems apparent that it is not easy to improve mathematics teaching. Many researchers and educators have believed that the quality of mathematics teaching can be improved by changing the traditional teaching practice. As implied above, the findings of research suggested that mathematics teaching practice could be to some degree changed by improving teachers' understanding about

mathematical concepts and representations.

The general goal of the current study was to better understand preservice elementary mathematics teachers related to their knowledge and representation about fraction concepts for supporting their mathematics teaching practice aligned with the reform-oriented way. Any improvement of our understanding about what preservice elementary teachers really need regarding mathematics teaching will stimulate professional researchers and teacher educators to continually seek a better model of mathematics teaching practice for preservice and inservice teachers as well as for schools.

THEORETICAL BACKGROUNDS

A long and widely held assumption about elementary school mathematics has been scarcely challenged by teacher educators, researchers, and policymakers in Korea. That is, the majority of them seemed to think that if you can correctly calculate problems from elementary mathematics to figure out answers, then you can teach them. Despite this wrong assumption held by people, teaching elementary mathematics for understanding is never easy. Especially, fractions are considered as one of the most difficult areas both for teachers to teach in a meaningful way and for students to learn throughout the elementary mathematics curriculum.

Among many contributors to our understanding about fractions, the views established by Kieran (1976, 1980) have attracted the most attention from researchers. According to Kieran, fractions consist of five subconstructs — part-whole, measure, quotient, operator, and ratio. He noted that a full understanding about fractions requires an understanding of the concepts of each five fraction subconstructs as well as the interconnectedness among them.

In order to find out features underlying fraction concepts, Pitkethly and Hunting (1996) reviewed a plenty of studies about fractions. As a result, they found out that partitioning schemes using continuous or discrete models and identifying the unit are fundamental to students' development of fraction concepts. The partitioning schemes are important for strengthening the knowledge about fraction concepts. Fractions as part-whole relationship have been first introduced in the elementary curriculum and used as the primary meaning for teaching and learning (Lamon, 2001). However, Pitkethly and Hunting reported that overemphasis on fractions as part-whole using continuous models tend to limit students' development of other fraction concepts.

As was noted earlier, many reform documents such as NCTM's *Principles and Standards for School Mathematics* (2000) demand for teachers to have representation

abilities to organize and communicate mathematical ideas. According to Shulman (1986), representation is regarded as one of the most effective forms of teachers' pedagogical content knowledge. Since Shulman, studies about representations have been rapidly growing in mathematics teaching and learning. Representation is a picture of the images on mathematical problems and generally takes a variety of forms including tables, diagrams, graphs, sketches, and words. In particular, forms of representations are classified as continuous or discrete models to reflect fraction concepts. Representation is a useful tool for mathematical thinking, communication information and mathematical ideas, and helping students understand mathematical concepts (Fennell & Rowan, 2001; Greeno and Hall, 1997). Greeno and Hall pointed out, however, that teaching the forms of representations in themselves should not be the goals of teaching and learning.

RESEARCH METHODS

A total of 115 preservice elementary teachers (30 male and 85 female) participated in this study. They all are in the third year of a 4-year elementary teacher education program at Gwangju National University of Education, S. Korea. Prior to the start of the present study, they took one mathematics course and one elementary mathematics teaching method course.

The instrument used in this study was focused on examining preservice elementary teachers' understanding about fraction concepts and their representation ability of fractions. The items covered the five subconstructs about fraction concepts as follows: part-whole, measure, quotient, operator, and ratio. For instance, preservice elementary teachers' understanding about a fraction as part-whole and their representation ability were measured by asking them to give a story problem for a fraction 3/4 used as part-whole and to generate an appropriate representation.

For analysis of preservice elementary teachers' understanding about fraction concepts, their responses were coded in two ways: (0) incorrect answer and (1) correct answer. On the other hand, their representation abilities about fraction concepts were analyzed by coding their responses in three dimensions: (0) unable to generate representation, (1) inappropriate representation, and (2) appropriate representation. Before analyzing the data, Cronbach's alpha coefficient was calculated to measure reliability for the instrument, and the result produced the satisfactory internal consistency of α=.69.

RESULTS

Table 1 shows that preservice elementary teachers were most familiar with fractions as part-whole, and least familiar with fractions as operator. About 69% of them (79 out of 115 respondents) were able to create examples of a fraction 3/4 used as a meaning of part-whole. A representative example of the responses is like “3 parts out of the whole partitioned into 4 parts of equal size.” However, about 30% of them failed to give an example appropriate to a part-whole fraction 3/4. Similarly, about 63% of preservice elementary teachers were able to create story problems appropriate to a fraction 3/4 as measure. As shown in the table, however, only about 23% of them (27 out of 115) were able to correctly explain the meaning of a fraction 3/4 used as operator by making up word problems. Meanwhile, the table shows that only about 43% of preservice elementary teachers successfully came up with story problems of a fraction as quotient and ratio. In addition, it was found that only 27 out of 115 preservice elementary teachers understood the meaning of fractions as operator.

These findings indicate that preservice elementary teachers have significant difficulty in understanding the concept of fractions as operator. In effect, it was not surprising that preservice elementary teachers have a lack of understanding about fraction concepts as ratio, quotient, and operator, because school curriculum and current teaching practices about fractions have most emphasized fractions as “part out of whole” rather than equally emphasizing the other views toward fractions. Interestingly, the overemphasis on fractions as part-whole is also found out in Korean students’ development of fraction interpretations. According to Kwon (2003), for instance, it was revealed that more than 90% of Korean elementary students tend to interpret fractions primarily as part out of the whole, but no responses were found for fractions as operator. This implies that students’ understanding about fraction concepts is strongly connected to teachers’ knowledge about fraction concepts and teaching practice. Thus, teachers’ limited understanding about fraction concepts may lead to the limited opportunities for students to explore various notions about fraction concepts.

Table 1. Preservice Elementary Teachers’ Knowledge of Fraction Concepts (N=115)

	Meanings	Representations		
		0	1	2
Part-whole	79	3	26	86
Measure	72	13	13	89
Quotient	49	25	48	42
Operator	27	43	58	14

Ratio	51	30	40	45

As was mentioned in the previous section, teachers' representation knowledge is closely related to teaching practice in classrooms and students' learning. Table 1 presents the frequencies of preservice elementary teachers' responses to representation questions on each of the five subconstructs of fraction concepts. The results indicate that more than 75% of them were able to successfully generate appropriate representations of fractions as part-whole or measure. In doing so, continuous models were most frequently used to represent a fraction 3/4 as part-whole or measure. For instance, fraction circles were the most frequently used too for representing fractions as part-whole, while number lines and rectangles were most preferred to represent fractions as measure. In a few responses, drawings were used to represent the real world context appropriate to a fraction 3/4.

When a fraction 3/4 is used as measure, it means three parts out of four equal parts of the unit. It is noteworthy to indicate that many preservice elementary teachers have quite difficulty in identifying the unit and distinguishing between the whole and the unit. For instance, when asked to place a fraction 5/6 between 0 and 2 on a number line, they tended to place the fraction between 1 and 2. This representative example of incorrect answers explains that they were confused the concept of the unit with the whole. That is, they seemed to consider a fraction 5/6 used for measure as five parts out of six equal parts of the whole rather than as 5(1/6) of the unit. With respect to preservice elementary teachers' incorrect notions about the unit, this finding implies that their overreliance on the concept of fractions as part out of the whole seems to make them difficult in understanding the concept of the unit in fractions.

In addition, the results of this study shown in the table revealed that the percentage of preservice elementary teachers' successful representations was significantly decreased for the other fraction concepts such as quotient, operator, and ratio. Only fourteen out of 115 respondents were able to generate appropriate representations about a fraction 3/4 as operator. Similarly, only about 35% of the respondents were able to appropriately represent fractions as quotient and ratio using tables or number lines. Additionally, the results shown in the table show that about 43 out of 115 respondents were unable to generate a representation of a fraction 3/4 as operator at all.

In sum, the findings of the present study indicate that the majority of preservice elementary teachers understood the notions of a fraction as part-whole and measure, and they also successfully generated appropriate representations of such fraction subconstructs using mostly continuous models such as fraction circles and number lines.

Meanwhile, the results showed that they have the significantly limited knowledge about the notions of fraction concepts as quotient, operator, and ratio as well as about representations of such fractions. This result implies that most Korean preservice elementary teachers still tend to view fractions as part out of the whole, although other conceptions of fractions need to be equally likely emphasized in school mathematics. The results of correlation analysis (r=.623) supported that the depth of preservice elementary teachers' understanding about the meanings of fraction concepts is strongly related to their representation abilities about fractions using continuous or discrete models.

CONCLUSIONS AND IMPLICATIONS

The study reported here was carried out by an attempt to examine the depths of Korean preservice elementary teachers' understanding about fraction concepts and representation abilities. To do so, they were asked to create appropriate story problems for a fraction 3/4 used as five different meanings — part-whole, measure, quotient, operator, and ratio — and represent it using pictorial models.

Pitkethly and Hunting (1996) pointed out that some research findings are dependent upon cultural contexts, such as educational system or textbook emphasis. In Korea, a fraction is first introduced as the meaning of part-whole concept in the third grade and explored through activities such as paper-folding or partitioning of circles and rectangles into equal parts. The view of fractions as part-whole is most emphasized throughout the elementary curriculum. The findings of the present study revealed that the Korean preservice elementary teachers' primary view of fractions as part out of the whole seems to be closely related to the educational context in elementary mathematics such as curriculum, teaching and learning.

Another goal of this study was to examine preservice elementary teachers' representation abilities about fraction concepts. Mathematics educators and researchers maintained that teachers' knowledge of representation plays a key role in mathematics teaching practice in classrooms and necessary for helping students understand mathematical ideas in a meaningful way. This study revealed that more than 75% of preservice elementary teachers were able to generate appropriate representations of fractions as part-whole or measure, while most of them had big difficulty in generating appropriate representations on fractions as operator. This feature about preservice elementary teachers' representation ability is consistent with the results of their understanding about fraction concepts as mentioned earlier. In effect, the correlation analysis between knowledge about fraction concepts and representations supported that

representation abilities are greatly influenced by the depths of their understanding about the notions of fraction concepts.

In addition, the findings of the present study implies that preservice elementary teachers' profound lack of conceptual understanding about fractions used as quotient, operator, or ratio may lead to overemphasize fractions as part-whole or measure using continuous models. As Pitkethly and Hunting (1996) noticed, "overreliance on the continuous part-whole model" can limit students' opportunities to explore and develop other fraction interpretations (p. 11). As suggested in previous studies, teachers' deep understanding about what fractions really mean and about how fractions are represented with models is required to teach mathematics in a way suggested by reform documents. Without understanding and representation abilities about various fraction concepts, teachers are expected to have difficulty in transitioning their traditional way of mathematics teaching into a reform-oriented way.

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