THE CALCULUS OF ENGINEERING AND MATHEMATICS UNDERGRADUATES[1]

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ABSTRACT

This paper investigates first year undergraduate mechanical engineering and mathematics students' conceptions of the derivative and the role that students' departmental affiliation might play in developing these conceptions. The findings reveal that mechanical engineering students develop a tendency for rate of change aspects of the derivative while mathematics students develop a tendency for tangent-oriented aspects. The data also suggest that students' departmental affiliation is an important factor in their developing conceptions of the derivative in the sense that students privilege particular forms of the derivative.

INTRODUCTION

This paper[2] is mainly concerned with student's departmental affiliation and its possible influences on students' conceptions. Nasir & Saxe (2003) define affiliation as identification with a common cultural ancestry and distinctive cultural patterns. Their definition accords with the sense that I make use of it. I view students' departmental affiliation as their identification with distinctive patterns of a specific undergraduate department. With this regard this paper will mainly attend to the question: "Can departmental affiliation, e.g. being an engineering or mathematics student, be ignored in research on undergraduate students' understanding of calculus?". My research strongly suggests that the answer is 'no'. I will outline my reasons for this answer throughout the paper.

Some research into undergraduate students' understanding of calculus appears to ignore the departmental affiliation of the students. I briefly outline two such studies that have commonalities with my work. Asiala, Cottrill, Dubinsky, & Schwingendorf (1997) conducted a study on 41 engineering, science and mathematics students. Their aims were to investigate learning about the slope of a graph of a function, to evaluate the extent that their Action-Process-Object-Schema (APOS) theoretical perspective

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[2] This paper has commonalities with a paper I submitted to PME 2004 but has a different focus. In my PME paper I attend to the issue of student identity. In this paper my focus is on students' departmental affiliation and its possible influences on students' understanding of the derivative concept.

was useful for understanding the mental constructions made by students, and to compare the performance of 17 students who took a reform-based calculus course with 24 students who took a traditional calculus course. Their focus was on the contribution of the different instructional treatments to students' conceptions of the derivative. They simply ignored the possibility that the departmental affiliation of the students might have influenced their conceptions.

The second study is that of Bezuidenhout's (1998) which explored university students' understanding of rate of change. A total number of 523 first year students from engineering, physical science, and service calculus (the commercial and management sciences) courses took part in his study. A random sample (100 students) were taken from these three different groups for the analysis of the results. He only presented findings about students' difficulties with understanding of the rate of change concept. Like Asiala et al. (1997), Bezuidenhout also disregarded the departmental affiliation of the students. Such studies appear to assume that departmental affiliation has no bearing on cognition.

Some research, however, considers that departmental affiliation may influence students' conceptions (see, Maull & Berry, 2000; Kent & Stevenson, 1999; Sazhin, 1998). Maull & Berry's study is particularly interesting from the point of view of departmental affiliation. They investigated first and final year mechanical engineering and mathematics undergraduates alongside postgraduate students and professional engineers. They concluded that "the mathematical development of engineering students is different from that of mathematics students, particularly in the way in which they give engineering meaning to certain mathematical concepts" (ibid, p.916). They noted that both groups of students showed similar patterns of responses at entry but, by the final year, the groups' responses diverged. They did not, however, provide reasons for this emergent divergence and called for further research.

THE RATIONALE OF THE STUDY

The study presented in this paper is part of a larger study exploring whether there is any difference between mechanical engineering (ME) and mathematics (M) students' conceptual development of the derivative concept over the first year and, if there are differences, to explore reasons for these differences. This study was carried out in a large university in Turkey. It is not argued that results from this study generalise beyond the confines of this university. The approach to data collection was naturalistic (Lincoln & Guba, 1985). Data were collected by a variety of means: quantitative (pre-, post- and delayed post- tests), qualitative (questionnaires and interviews) and ethnographic (observations of semester 1 calculus courses and 'coffee-house' talk).

In order to give the reader an appreciation of the development of the rationale of the study, summary data on classroom observation of calculus courses and on pre-, post- and delayed post- tests will be presented.

Both calculus courses were taught by mathematics department lecturers. The calculus courses were observed and compared with students' notes to gain insights into which aspects of derivative were 'privileged' in each department (Wertsch, 1991, p.124 uses 'privileging' in place of 'domination' to emphasise that a mediational means may be viewed as most appropriate in a particular setting). The analysis of observations and students' calculus course notes indicate (see table 1) that ME students were taught more application (rate of change) aspects of the derivative compared to M students who were taught more theoretical or tangent-oriented aspect.

	Rate of change		**Tangent**	
	ME	M	ME	M
Duration **Examples**	≈133 minutes (9 examples)	≈11 minutes (no examples)	≈10 minutes (no examples)	≈85 minutes (7 examples)

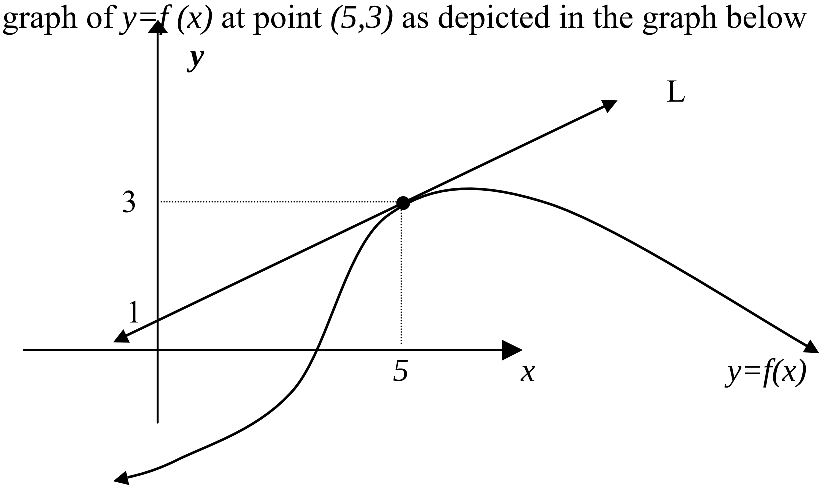
Table 1: The general analysis of ME and M calculus courses' notes

The pre-, post- and delayed post- tests were administered to 50 first year ME degree and 32 M degree students and addressed questions regarding, 'rate of change' and 'tangent' and were used to gain insights into: (a) how ME and M students' concept images of the derivative developed over the course, (b) how students dealt with rate of change and tangent concepts when questions were presented in graphic, algebraic and application forms and (c) whether there were any differences between ME and M students' performance in the different forms of these questions.

The pre-test was given to all students at the beginning of the course and there was, in general, no significant difference between ME and M students' performance. Both groups of students improved in their performance in the post-test which was set at the end of the first semester. The delayed post-test was set towards the end of the second semester and both groups of students continued to improve. ME students, overall and in comparison with M students, did much better on rate of change-oriented test items regardless whether the items were presented in algebraic, geometric or application-based forms in the post-test. Similarly M students did much better than ME students on all forms of tangent-oriented questions. This trend (ME to rate of change, M to tangent) remained strong in the delayed post-test.

As an example, two questions (one tangent-oriented and one rate of change-oriented) administered to all students in all three tests together with the obtained results are presented below. The purpose of providing these two questions is to give the reader an appreciation of students' development of conceptions from the pre-test to the delayed post-test.

Question 1: Line **L** is tangent to the graph of $y=f(x)$ at point $(5,3)$ as depicted in the graph below



a) $f(5)=?$
b) $f'(5)=?$
c) What is the value of the function $f(x)$ at $x=5.08$ (be as accurate as possible)?

Figure 1[3]: Question 1

Categorisations of Responses	Q1-b						Q1-c					
	Pre-Test		Post-test		DPT		Pre-Test		Post-test		DPT	
	ME	M	ME	M	ME	M	ME	M	ME	M	ME	M
C	22	25	32	42	42	56	10	18	4	34	10	34
PC	34	22	16	19	4	13	46	44	46	41	36	22
IC	30	44	44	37	32	22	6	16	16	3	26	16
NA	14	9	8	3	22	9	38	22	34	22	24	28

Table 2 :Students' responses (percentages) to question 1

Codes: C-Correct, PC-Partial Correct, IC-Incorrect, NA-Not Attempted, DPT-Delayed Post-Test

NB: Q1-a is not presented as all students obtained the correct answer in all three tests.

Question 2: Find the rate of change with respect to the given variable of the following functions at the values indicated.
a.) $f(x)=x^2-7x$, when $x=3$
b.) $g(x)=(x^2-1).(x+1)$, when $x=1/3$

Figure 2: Question 2

Categorisations of Responses	Q2-a						Q2-b					
	Pre-Test		Post-test		DPT		Pre-Test		Post-test		DPT	
	ME	M	ME	M	ME	M	ME	M	ME	M	ME	M
C	24	22	84	53	74	53	18	22	64	53	76	50
PC	10	22	2	13	8	19	8	19	18	13	6	19
IC	56	50	12	25	16	22	54	50	16	25	16	25
NA	10	6	2	9	2	6	20	9	2	9	2	6

Table 3: Students' responses (percentages) to question 2

[3] It should be noted that my question 1 is almost identical to Asiala et al.'s interview question 6.

For question 1b & 1c M students outperformed ME students, especially in question 1c, by providing more correct responses in the post- and delayed post- tests. For question 2 ME students this time outperformed M students by providing more correct responses in the post- and delayed post- tests. Since similar results and trends (ME to rate of change, M to tangent) were found in other items of the tests as well, I decided to explore this issue (trend) in depth.

EXPLORING THIS PHENOMENA IN GREATER DEPTH

The results show a clear trend: ME students do better on rate of change-oriented items whilst M students do better on tangent-oriented tests' items. They do not, however, reveal why this trend exists. To explore this matter further I designed two additional items (see Figure 3) which might shed further light on the reasons behind this trend, and administered them after the delayed post-test . The remainder of this paper focuses on students' responses to these two items.

Item 1: If the following two questions (A and B) were given in an examination and you only had to solve one of them, which one would it be? Please tick just one option and explain why you would choose that one.

A.) At a certain time (*t*, seconds) the rate at which water flows (m^3 /sec) into a water tank is given by the formula $f(t)= t^2/4 +24t+125$. Find:

a.) The initial amount of water in the tank and its initial rate of change?

b.) What is the rate change of flowing water at any time, *t*?

c.) The time at which the rate of change is $32\ m^3/sec^2$

B.) Find the solutions of the following questions:

a.) Verify that the gradient of the tangent to the curve $y=x^2$ curve at a point $(x_1, x_1^2)=2\ x_1$.

b.) Find the equation of the tangent to the curve $y=2x^2-x+3$ which is parallel to the line $y=3x-2$.

c.) Show that the graph of $f(x)= x^{1/3}$ has a vertical tangent line at (0,0) and find an equation for it.

Item 2: Two university students from different departments are discussing the meaning of the derivative. They are trying to make sense of the concept in accordance with their departmental studies.

Ali says that "Derivative tells us how quickly and at what rate something is changing since it is related to moving object. For example, it can be drawn on to explain the relationship between the acceleration and velocity of a moving object".

Banu, however, says that "I think the derivative is a mathematical concept and it can be described as the slope of the tangent line of a graph of *y* against *x*".

a.)Which one is closer to the way of your own derivative definition? Please explain!

b.) If you had to support just one student, which one would you support and why?

Figure 3: Two items to explore reasons for rate of change and tangent orientations

RESULTS

I first present quantitative data (frequency counts) and then a categorisation of students' reasons for their choices. Table 4 & 5 show students' responses to items 1 and 2.

Item 1	ME	M
Question A (A)	60 % (27)	22 % (7)
Question B (B)	40 % (18)	78 % (25)

Table 4: Students' responses (percentages–raw frequencies in brackets) to items 1

	ME		M	
Item 2	Item 2a	Item 2b	Item 2a	Item 2b
Ali (A)	51 % (23)	49 % (22)	19 % (6)	13 % (4)
Banu (B)	27 % (12)	49 % (22)	63 % (20)	78 % (25)
Both (A &B)	22 % (10)	2 % (1)	16 % (5)	3 % (1)
Not Attempted (NA)	0	0	3 % (1)	6 % (2)

Table5 Students' responses (percentages–raw frequencies in brackets) to items 2

For item 1 ME students demonstrate a preference (60:40) for rate of change-oriented question over tangent-oriented one whilst M students show quite a strong preference (78:22) for tangent-oriented question. Similar preferences can be seen in item 2a. In item 2b the preference of the M students remains the same but the ME students are equally divided.

Tables 6 and 7 address a categorisation of students' reasons for their choices in items 1 and 2. Repeated reading of students' responses generated three emergent categories: Affiliation; Practice; and Ease. 'Affiliation' (see Nasir & Saxe, 2003) is concerned with identification with a common cultural ancestry and distinctive cultural patterns. 'Practice' in this paper concerns students' calculus practices and, as these students are novice practitioners, is related to what goes on in calculus courses. 'Ease' here means what particular students reported that they found easy (not my decision on the ease of items). I first explain how I allocated students responses to these categories, and give examples of students' response for each category.

Affiliation (ME) ME students' responses were placed in this category when they mentioned any of the following: real life; applications; rate of change; engineering. For example;

Student 1:'*Ali's derivative interpretation is closer to me because Ali is able to make the derivative concept concrete through giving us an example of a physical event which we can encounter in real life. Banu sounds to deal with derivative concept on the paper but derivative shouldn't be just a mathematical concept; an engineer should be able to carry the derivative from the paper dimension to 3-dimensional space context'*.

Affiliation (M) M students' responses were placed in this category when they mentioned any of the following: the exact nature of the derivative; the slope of a tangent; belonging to a mathematics department; interpretation from a mathematician standpoint; the comprehensiveness of the definition.

Student 2:'*Banu's (derivative interpretation) is closer to me because she is a mathematician like me'.*

Practice (ME) ME students' responses were placed in this category when they mentioned the way calculus is being covered and used in their department.

Student 3:*'We are using it in that way and learning it that way'.*

Practice (M) M students' responses were placed in this category when they mentioned not knowing much about rate of change or the way calculus is being covered in their department.

Student 4:*'We are learning in that way and I don't know much about the rate of change'.*

Ease(ME & M) Students' responses were placed in this category when they mentioned the 'ease' of this way of thinking about the derivative.

Student 5:*'Because it is easier'.*

Categorisation of responses	Engineers choosing A			Mathematicians choosing B		
	Item1 60% (27)	Item 2a 51% (23)	Item 2b 49% (22)	Item1 78% (25)	Item 2a 63% (20)	Item 2b 78% (25)
Affiliation	20% (9)	44% (20)	44% (20)	13% (4)	47% (15)	66% (21)
Practice	11% (5)	7% (3)	4% (2)	50% (16)	28% (9)	13 % (4)
Ease	29% (13)	0	0	16% (5)	0	0

Table 6: Responses (percentages–raw frequencies in brackets) of ME students who chose As and M students who chose Bs

'Affiliation', 'practice' and 'ease' are cited by both groups of students for item 1 and there is no clear pattern to these responses but note that 50% of M students cite 'practice'. 'Ease' is not really applicable for item 2 and all students cite either 'application' or 'practice' with 'affiliation being the dominant stated reason.

Categorisation of responses	Engineers choosing B			Mathematicians choosing A		
	Item1 40% (18)	Item 2a 27% (12)	Item 2-b 49% (22)	Item1 22% (7)	Item 2a 19% (6)	Item 2-b 13% (4)
Affiliation	0	18% (8)	49% (22)	6% (2)	19% (6)	10 % (3)
Practice	9% (4)	9% (4)	0	0	0	0
Ease	31% (14)	0	0	13% (4)	0	0
Not categorised	0	0	0	3% (1)	0	3% (1)

Table 7: Responses of ME students who chose Bs and M students who chose As

'Ease' is the dominant cited reason in item 1 and 'affiliation' is the dominant cited reason in item 2 for students who do not follow the 'ME – rate of change, M – tangent' trend. That 49% of ME students' affiliation (item 2b) is with a mathematician's concept of the derivative is noteworthy and, perhaps, a little unexpected.

DISCUSSION

On the basis of the analysis of presented data, 3 questions are addressed: Do students' responses to items 1 and 2 shed light on the trend 'ME to rate of change, M to tangent' apparent in the post and delayed post-tests? Do students' departmental affiliation influence students' conceptions of the derivative and if so, then what can be said about the relationship between students' conceptions of the derivative and their departmental affiliation? How do students' departmental affiliation develop and what can be said about this development?

Calculus courses that students follow appear to be one of the reason for the emerging trend of 'ME to rate of change, M to tangent'. Both ME and M students acknowledge the influence of calculus practices (see table 6) whilst explaining the reasons behind their choices of the forms of derivative. The analysis of observations and students' calculus course notes also indicate (see table 1) that ME students were taught more application (rate of change) aspects of the derivative compared to M students who were taught more theoretical or tangent-oriented aspect. Calculus practices in each department are thus likely to have played a significant role in the emergence of this difference. In this regard, Kendal & Stacey (2000) illustrate how students' conceptions of the derivative are strongly affected by the aspect of the derivative privileged by their teachers. It is hence reasonable to infer that privileging of rate of change and of tangent aspects of the derivative in the two calculus courses influences students' orientation and knowledge development.

Can calculus course practices alone account for ME students' tendency to rate of change and M students' tendency to tangent aspect of the derivative concept? From Asiala et al. (1997) and similar studies' point of view, the answer would probably be 'yes'. Such studies are, in general, conducted to demonstrate the 'effectiveness' of their instructional treatments. From their perspectives, it is the nature of instructional treatments that largely brings about diversity between the conceptions of students of different groups or 'better understanding' in favor of one group. However, in my study students' responses suggest that the practices alone are not sufficient to account for the different tendency between the two groups. As tables 6 and 7 demonstrate, affiliation is the highest cited reason by both groups of students in explaining their preferences of the forms of the knowledge. I think that the stance of centralising the role of the nature of practice in explaining students' conceptions at school level is largely justifiable but at university level the situation is more complex. At university level, students, in general, already made conscious decisions and opted for mathematics, engineering or other subjects to study. As could be seen from the Student 1's extract, students are more conscious of what they learn or what they should learn. Their tendency or preferences of what they learn can be influenced by their association with department in which they study. Therefore, it can be posited that affiliation is likely to have played a crucial role in the emergence of the difference between the two groups.

Affiliation is one of the three categories that repeated reading of students' responses generate. Its generation builds on each group of students' responses for their preferred forms of knowledge that show their developing affiliation towards particular forms of derivative conceptions. For instance the applicability of the derivative concept is deemed as being very important by ME students and this is one of the central reasons behind their choices of rate of change-oriented item and hence developing affiliation towards it. On the other hand, M students seem to attach considerable importance to the 'mathematicality' of the derivative concept and demonstrate the tendency for this form of the derivative. It is thus plausible to infer that the reasons behind the students' different inclinations for the different aspects of derivative are not only because their practice directed them in that way but also because students of each department have developed a different affiliation towards different aspects of the derivative.

What now can be said about the relationship between students' conceptions of the derivative and their departmental affiliation?. Students seem to build a relationship with particular forms of the knowledge in accordance of their departmental perspectives. For example, engineering students have proclivity to forge an association with practical aspect of the derivative concept as they consider engineering as a practical profession (see, student 1). Mathematics students, on the other hand, tend to build a relationship with the 'mathematicality' of the derivative concept and see this tendency as being consistent with belonging to mathematics department (see, Student 2). That is why, I think that there is a parallelism between each group students' affiliation and their developing conceptions of the derivative concept.

Since the results suggest that students' departmental affiliation influence their conceptions, this issue deserves close scrutiny. Some important questions arise: How do students' departmental affiliation come into being or develop? Does the development of students' departmental affiliation occur after they embark on their studies at university? Does its development evolve with the time and the process of this development goes back to earlier stages of their education? Since data analysis is ongoing, I cannot speculate on the development of students' sense of affiliation before the university stage. I confine my discussion regarding this issue to university level and shortly attend to it.

The two departments have different implicit goals not only with regard to calculus but also with regard to whole structure and practices of the community/department itself. At the observable level, this is manifested in the types of examples that each department privileges (see Table 1). Therefore it is likely that students' developing affiliations will, in general, be influenced by their interpretation of these implicit goals. This impacts on how students locate and 'position' (Holland Lachicotte, Skinner, & Cain, 1998) themselves in terms of which aspect of the concept to learn and privilege. Students' developing affiliation towards particular forms of the knowledge may then come into being as a result of the way that students comprehend and enact their positions in the department to which they belong. In the course of their studies, then it appears that students of each department begin to position themselves according to their professional perspective and this positioning influences their proclivity towards forms of knowledge. As a result, students' sense of affiliation and the way that they position themselves may play an important role in developing students knowledge and conceptions

CONCLUSION

Mechanical engineering students develop a proclivity for rate of change aspects of the derivative whilst mathematics students develop a proclivity for tangent-oriented aspects. Further to this, it appears that students' conceptual development of the derivative and the way they build relationship with its particular forms are closely related and co-evolve in accordance with the departmental perspectives.

It has been argued that this difference between the conceptions of the both groups students cannot solely be attributed to the practice of the courses that the students followed. Departmental affiliation appears to have influence on cognition and play a crucial role in the emergence of this difference. It is thus suggested that research into different departments students' conceptions of subjects at the university stage pay attention to students' departmental affiliation.

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