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An Investigation of the Cognitive Processes required for a Mathlet

Regardless of the simplicity of the mathlets, there is an array of cognitive processes a student needs to develop and use to make effective use of mathlets. A student first would need to visualize the mathlet to see both the static diagrams at any second, but also to understand the notion of dynamic geometry, interact with the mathlet to fully utilize its representation components, and identify the individual parameters to make use of them in meaningful manner. Then the student would need to analyze the mathlet to identify the underlying mathematical relationships, interconnected representation systems- synthesize all the parts to make a conjecture, and investigate the emerging patters with where each pattern leads to. The student would finally use these patternistic understanding to intuit for similar and different kind of mathlets. But also would need to use this insight for further explorations of the same topic under consideration.

Sometimes student would need to judge the correctness of the representation in a mathlet and also identify the restrictions of a mathlet so as to make appropriate and accurate generalizations. Accurate drawings are needed for correct reasoning according to Kortenkamp & Richer –gebert (1998). They mention Interactive Geometry Systems (ICGs), and argue that these enable students to consider drawing as a dynamic entity. With dynamic modeling, dragging, macro-level repetitive tasks, and locus (tracing the movement of a point) students can identify and perform mathematical or geometrical tasks.

Benett (1997) argues that, we must not underestimate the power of dynamic geometry for simply creating good static ones. Besides supporting visualization, these static variations of geometrical figures as well as their interconnectedness, may enhance our understanding of geometry and students' thinking with different representations. Balacheff & Kaput (1996) argue that dynamic visualization is a new experiential realism. They state that this change is occurring at the level of mathematical experience. Students would need to shift their experience with mathematics to a more experiential level in which they can use the parameters and representations to be manipulated as inputs and the changes on the screen as outputs of a mechanism.

Jones (2001) uses the term "functional dependency" for the context of dynamic geometry referring to the robustness of a figure under drag. We can use his terminology to include different representation systems inside an Id which are dependent upon other representation systems and which function together. Here Jones states, "the things that do not move are the geometrical relationships that have been constructed" (p.72). Hence, the student would need to analyze an ID to identify these geometrical relationships.

Mathlets as mathematical representations; the role of insight

Any mathlet can be thought as a mathematical representation of a geometrical or mathematical relationship as well as a composed representation of a combined group of mathematical representations as graphical, tabular, symbolic (formulae), pictorial (as long as the mathematical characteristics of specific shapes are conserved) or even verbal (Goldin, 2002; Cezikturk, 2003). These types of representations are also named as "external representations" other than the "mind's eye" internal representations of students (Tabachneck-Schijf, Leonardo & Simon, 1997). Palmer (1997, cited in Kaput, 1987) states that every mathematical representation has two parts; a representing world, and a represented world. These two worlds function separately from one another as separate systems, in the form of a mathlet and its corresponding mathematical concept or entity. Children' ability to make the connections between the represented world and the representing world as well as among many types of representing worlds is directly linked to the development of their intuition or insight (Janvier, 1987). In other words, children can grasp mathematical representations if their intuitive understanding is enhanced. Remembering the babies and how they learn, children who are exposed to more representations would be less vulnerable to different examples of representations. It is a two-sided coin; students need intuition to make sense of representations but also use of representations help them develop intuition.

Use of multiple representations are frequently linked to the development of insight and intuition of students. This idea is supported by the studies of Tabachneck et.al. (1997), Behr, Harel, & Post (1992), Cheng (1999) and Brenner et al. (1997). Sternberg and Ben-Zeev (1996) argue that any representation is restricted in some information and excess of some other information. Using multiple representations would help to reduce these imbalances and build better cohort examples. Adiguzel & Akpinar (2001) concluded that computer based instruction rich in multiple representations improves students' performance and provides them with the cognitive tools needed to develop problem solving skills.

Van Hiele Theory and Insight

As a theory of understanding structures, van Hiele theory emphasizes the role of insight on learning mathematics. According to van Hieles, a person who acts with insight acts with respect to the structure he or she perceives rather than acting at random (van Hiele, 1986). Facts may be lost forever, however the remaining part of the structures facilitates recall of the forgotten. Hence, each new structure is understood up to the degree the previous structures are understood (van Hiele, 1986). This learning may be promoted by visual aids (Rambally,1982; Thompson, 2002)and computers may be a good reference for achieving this (Bennett ,1997). Instantaneous and precise variations are key words while talking about the graphs driven from interactive programs. Bennett continues with stating that whatever value dynamic geometry would have, we must not underestimate it for simply creating good static figures.

Representations, van Hiele Levels, mathlets and translations

A good representation should lead the learner to the overall topology of the networked concepts and some means to indicate where in the structure any particular concept belongs (Cheng, 1999). A representation is a mapping between two structures or two worlds as any representation in a mathlet as much as the mathlet itself would be. Most mathlets enables the student to simply change the direction of this mapping and see the changes in the original structure with parameter manipulations within the second structure while the arrow of the mapping is now reversed. Here, the student can be ready for either or both of the mappings (translations). According to van Hiele, this would be related to the familiarity of the structures to the student and also the students' readiness level as in the terms of van Hiele levels. If the first or second mapping would be counter intuitive to the student (i..e if the student has not been exposed enough to the each structure an translation before), this shift would be highly difficult to make.

Cunningham (1991) points to identifying how the visual and symbolic representation systems complement each other. As much as different mode translations, same mode translations are necessary and important to make clear if the students' difficulty with translation is due to the nature of the translation itself rather than its mode. Sometimes, an indirect translation may replace two direct translation combinations due to its inviting nature. Any mathlet has two distinct characteristics; types of representations and the nature of translations involved. As a metaphor, or as a conceptual map in the form of an unfolded cube, mathlets may enable the learner to see the structure of a task and familiarize with it.

Methodology

This is a part of a dissertation study, which also included some quantitative analysis. In this study, students' mathematical understanding inferred from their interactions with a selected mathlet on quadratics with three different and interlinked representations; tabular, symbolic and graphical affected by their van Hiele levels or by the nature of the translation was the main point. Three research questions were aimed to be answered:

1. How do students with different van Hiele levels experience mathlets?
2. How do notions of "structure" and "insight" as defined by van Hiele and Janvier function for students with inbetween van Hiele levels when students are working on translations among different modes of representation in mathlets that review quadratic equations content?
3. How do students process same mode and different mode translations while working with an interactive diagram?

The qualitative approach was used to see if students with inbetween van Hiele levels experience Interactive Diagrams differently from the students with high and low van Hiele levels. 17 students (6 high, 6 low and 5 inbetween van Hiele levels) from a public school in upstate NewYork established the subjects of this part of the study. These students were selected from their classes randomly reflecting three levels and the four classes used in the remaining part of the study. These four classes were two classes of two mathematics teacher who taught MATH AB and MATH ABh classes respectively. These two courses are same but for grade 9 and grade 10 students. The ethnic diversity of the school consisted of 18% Black students, 5.5% Hispanic, and 2.1% American Indian (The NewYork State School Report Card, 2002).

Instruments

Quadratic Functions in General Form Mathlet

This applet has three interrelated parts. One part for formulas functioning together for discriminant, axis of symmetry, and the half distance between a root of the quadratic function and the axis of symmetry. A slider for a,b,c coefficients acts as a tabular representation and forms the second part. The third part demonstrates the graphical representation of the parabola of the function specified with some hot points (the vertex, the x-intercepts and the arms of the parabola) being movable.

Interviews

Students are interviewed for their understanding with the Interactive Diagram. The interviews were administered by the help of the one trained interviewer during the lunch period. Videotaped interviews are done with these students whose van Hiele levels were already identified via Usiskin's van Hiele test carried. The students are given intervention with mathlets as a part of the whole study, which was a pretest-posttest quasi-experimental design. Interviews are carried out after posttests were administered in a one-week row. In the intervention phase, their teachers acted as facilitators and one to one lecturers for specific problems by students. In the intervention, students were required to fill out an activity sheet, in which they would find the vertex, x-intercepts, y-intercept, and min/max characteristics for a set of 10 quadratic equations ranging from y=x2 to y=3x2-2x+5. They were asked to rate their observations also. In the interviews though, they were not required to fill a worksheet rather, they were asked questions like; what different parts you see on the screen?, how do changes on different parts work together?, what happens when you change something here and see a change there?, what kind of changes occur within the graphic part only, etc. Some probing questions were also used when necessary; how did you get that?, can you point on screen where are you looking at?, etc.

Conclusions

Less confident inbetween students tend to use wording with less mathematical terminology than the high and low van Hiele students. They tend to ignore the representations and translations that they are not familiar with. They focus on various representation modes but they are highly reluctant to make translations to the other modes. Overall inbetween students seem to have the most problems with the translations between representations while they favor more of the same mode translations. A gear metaphor is established by the researcher to explain the situation with the interesting case of the inbetween students. The mathlet representation system is thought as a gear system in which some parts of the gear makes the while system move while does not affect movement in some other parts by itself. Intuitive and counterintuitive thinking was thought as a huge descriptor of this system yet not fully explanatory. In multiple representation systems the counter intuitive thinking is indispensable because each wheel or gear is used to make sense of the whole structure of the representation. Here the counter intuitive direction refers to the backward thinking required from a translation of which the reversed more intuitive way of thinking would be forged inside a normal classroom setting or in a textbook.