

DYNAMIC GEOMETRY SOFTWARE AS A SIMULATION TOOL FOR ALGEBRA PROBLEMS

– PROPOSAL –

Stefan Halverscheid
Universität Bremen, Germany

Dynamic geometry software is nowadays equipped with functions which go beyond features of purely geometric character. They have computer algebra elements at their disposal which provide a link between problems of algebraic nature and their geometrizations. Learning environments for the geometrization of algebra are presented in which students have chosen dynamic geometry software instead of computer algebra systems. With the help of these examples, the question as to how dynamic geometry software can be used as a simulation tool for algebra problems is discussed.

AN UNEXPECTED OBSERVATION

In the framework of a research project aiming at examining ways of teaching geometry within 'non-geometric' areas, several classes were given the task to work on the geometric side of certain algebra problems. In the first period of the project, I considered computer algebra systems as most appropriate among possible new media. Since learning processes were in the focus of the project, the decision was made to leave the choice as to which tool to take for what problem to every student individually.

In several classes - in which students are acquainted with the use of both computer algebra systems (CAS) and dynamic geometry software (DGS) - some of them, and quite often the best among them, have chosen a DGS program unlike my expectations.

In one session, for instance, students of 11th grade were asked to find a geometric criterion on when the system of equations

$$x^2 + y^2 = 1; \quad 3x + 2y = c; \quad c \in \mathbf{R},$$

has a solution. Student A discussed for two minutes with her colleague B with paper and pencil about the geometric objects involved here. Then, she went to a computer in the classroom and started some DGS, whereas B decided to ask a CAS for the algebraic answer at another computer.

Student A introduced a parameter c , whose value can be varied in the program by moving a point along a segment. Depending on the value of c she programmed the corresponding lines, computed the point of intersection and found out that the perpendicular to the line through the origin plays a particular role in this problem. The interviewer asked:

- | | | |
|---|---|--|
| 1 | I: Warum hast du nicht Maple gewählt? | I: Why haven't you chosen Maple? |
| 2 | A: Wär' auch OK gewesen. Es ist direkter so. | A: This would have been OK, too. It is more direct |
| 3 | | this way. |
| 4 | I: Was meinst du damit? | I: What do you mean by that? |
| 5 | A: Bei Maple muss ich Zahlen tippen eintippen | A: With Maple, I must type the numbers |
| 6 | und gehe zwischen Graphik und Text dauernd | and I keep going forth and back between |
| 7 | hin- und her. | text and graphics all the time. |
| 8 | Man ist hier direkt dabei, wenn 'ne Lösung | Here, you are directly with it when a solution |
| 9 | auftaucht. | appears. |

10 Bei Maple bin ich manchmal nicht sicher,
11 ich als nächstes tun soll. Bei EUKLID
12 probiere ich dauernd was.

With Maple I am not sure at times, what I
should do next. With EUKLID¹ I always try
something.

In A's statement, two features of CAS with graphics interface are mentioned which can be seen as a challenge for their use in the teaching of mathematics:

- These professional tools have achieved a level of sophistication which provides rich opportunities for building up internal representations of a certain setting. Theoretically, everyone with a reasonable background in the relevant area can produce a lot of examples and can get an idea of what is going on. It is up to the teacher to encourage the students to develop abilities to translate their observations in mathematical content. Moreover, how to work out a systematic investigation of some phenomenon, for instance, is not at all evident to every student.
- The way a graphics interface in a CAS works does not go without problems from a constructivist's point of view: Typing an equation and hitting the "ENTER"-key yields a picture whose shape can be modified. It is undisputed that these can be used to build up notions, intuitions and mathematical operations. Between the algebra problem and the geometry, however, there is a huge black box: It is not at all clear to a user of the system how the computer realizes the professional pictures.

Several discussions of this type brought up the question when and in what the use of DGS seems more appropriate than that of CAS. The point is surely not the quality of the graphics; in most examples, the picture's quality and their accuracy is higher in a graphics interface of a CAS. It is also much faster to use a CAS system to produce one picture.

LEARNING ENVIRONMENTS FOR VISUALIZING ALGEBRA WITH DGS

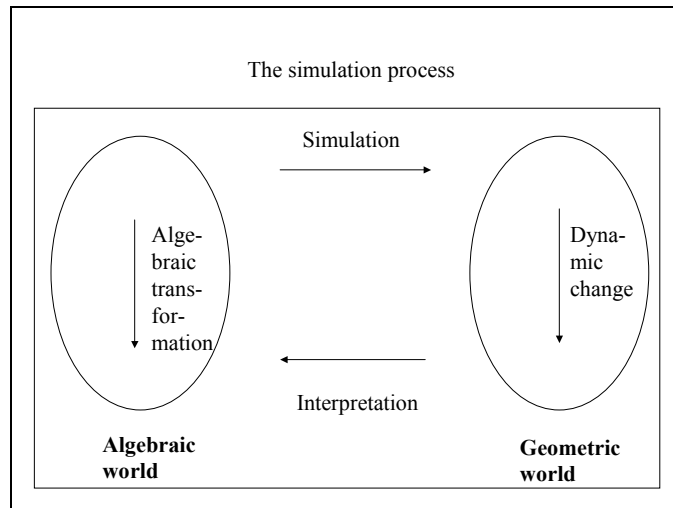
From this observation, it was tried at different grades (9, 11, 12 and university students, 2nd year) to build a microcosm CAS tool with a DGS. The following learning environments have been tried in the project in this direction:

- Linear equations: How to explain solution algorithms geometrically (9th grade)
- Quadratic equations: Stability of the solution set. (9th grade)
- Circles and lines (11th grade)
- Billiards: The trajectory of a ball on rectangular and elliptic tables. (12th grade)
- Three-dimensional linear algebra and analytic geometry: Visualizing planes and lines on the screen (12th grade, university students 2nd year)

SIMULATING ALGEBRA PROBLEMS WITH DGS

Let us discuss for a moment the situation in the geometrization of algebra. On the one hand, there is an algebraic problem, which should be solvable by certain algebraic operations. On the other hand, there is the geometric counterpart constructed from a setting in algebra. It is the aim of this project to understand more about the learning processes when students describe the algebraic operations as a dynamical change of the geometric setting.

¹ A DGS which is probably the most popular in Germany.



Students are all the time confronted with a permanent change of representations of their problem. The simplest example, perhaps, for different problems this change of representations can cause are the visualization of a system of two linear equations in \mathbf{R}^2 with no solution. Here the fact that DGS is not yet as elaborate as CAS gives students experiences with subtleties of a simulation process on an easy level.

If two linearly dependent and non-equivalent linear equations are given, the situation is simulated by two lines in the Euclidean plane. The students B and C have determined two points on each line algebraically and defined the lines in the DGS by these two points. DGS provides measuring instruments, which can have a bad surprise in store. Most DGS programs determine a point of intersection outside the range of the screen despite the expected parallelity.

If one of the lines is moved onto the other one, an interpretation in the algebraic world is often done by determining two points on the new line with DGS and by calculating the corresponding equation of the line. Unfortunately, this line is often independent of the other line, although the picture suggests that this cannot be. Let us listen to B and C after having this experience:

- | | | |
|------|---|---|
| 1 B: | Wozu ist ein Programm da, wenn es lügt? | What is a program good for, if it lies? |
| 2 C: | Es gibt alles. Computer lügen nich`. | It does its best. Computers don't lie. |
| 3 B: | Aber die Ergebnisse haben nichts mit | But the results have nothing to do with |
| 4 | der Wahrheit zu tun. | the truth. |
| 5 C: | Für absolute Sicherheit kann man ein | For absolute security, you cannot use |
| 6 | Computerprogramm nicht nutzen. | a computer program. |

In fact, these students take the perspective of philosophy of science. They seem to ask why one can attribute a function to a simulation, even though - as a theoretically based deduction - it cannot be a more substantial force than the theory itself.

The reason for this should be given - at least partially - in the heuristics which play a role for the simulation. Furthermore, the several realizations of theory and simulation have two important advantages: The concept of DGS links dynamic component and the iconic representations, which are at the same time criteria to distinguish description, formal structure and simulation. Mathematical equations structure the relations as constitutive entities of the system. In our context, but similarly in other areas like physics, they serve as structures of interpretation of a symbolic model and are - as such - fixated in the medium of the writing.

Simulations with DGS are instruments for the dynamical presentation of writing based structures, which are normally used in algebra.

SUGGESTIONS FOR DGS ELEMENTS WITHIN CAS

The development of algebra components within dynamic geometry software has certainly not reached the level of perfection which would be desirable. For certain geometric constructions involving decisions, leave alone for more algorithmic work it would be desirable to make if-then-decisions available for makro environments. As an example note that the realization of lines in the context of the theorem of two or three line reflections (as a composition of which a given congruence mapping can be written) cannot be programmed without such a decision.

It would be desirable to equip CAS with DGS elements to use the sophistication of the CAS together with the following DGS features:

- Instead of typing numbers – quite a discrete operation – it could be useful to enable the users to change the parameters continuously. Also the opportunity to create a short movie by changing a parameter is not optimal because the changes of parameters and the representation of the graphics do not happen simultaneously.
- A two-window technology – one for the algebraic side and the other for the geometric side – would be desirable such that both sides can be changed at all times and such that the other side performs a corresponding change automatically. Java scripts with this feature can be found on the web in several, but very special circumstances. For the change of representations, it would be desirable to give the opportunity to students to construct this in general situations themselves.

Selection of references:

Hoelzl, R. (2001). Using dynamic geometry software to add contrast to geometric situations – A case study. *International Journal of Computers for Mathematical Learning*. 6 (1): 63–86.

Laborde, C. (2001). Integration of technology in the design of geometry tasks with Cabri-geometry. *International journal of computers for mathematical learning* 6, pp. 283–317. Kluwer Academic Publishers.

Straesser, R. (2001). Cabri-Geometry: does dynamic geometry software (DGS) change geometry and its teaching and learning? *International Journal of Computers for Mathematical Learning*. V. 6(3) pp. 319-333.