Student's Development of Geometrical Concepts Through a Dynamic Learning Environment

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Geometry is one of the important areas of mathematics over the world. Geometry provides experiences that help students develop understanding of shapes and their properties. It enables students to solve relevant problems and to apply geometric properties to real-world situations. National Council of Supervisors of Mathematics endorsed that geometry was one of the ten proposed basis skill areas (NCSM, 1976) and is indeed a basic skill that should be taught to students of all ability levels (Sherard, 1981).

Technology is promoted and effective tool to teach and learn geometry. When technology is used appropriately, it can provide a rich environment in which students' geometric understanding and intuition can be developed (NCTM, 1989). Calculators and computers with appropriate software transform the mathematics classroom into a laboratory much like the environment in many science classes, where students use technology to investigate, conjecture, and verify their findings (NTCM, 1989).

One of the important vehicles of technological chance in geometry classroom is the use of Geometers' Sketchpad (GSP) (Jackiw, 1991). This software allows mathematics to be taught visually to the class as a whole, to small groups, or to individuals by creating dynamic and productive three way interaction between teacher, student, and computer (Hativa, 1984). GSP enables a student to "drag" part of configurations around and other parts of sketch automatically adjust. It enables students and teachers to investigate and construct unlimited geometric shapes. The shapes are first created and then they are explored, manipulated and transformed to ideal concept. Students cannot be creative enough in a traditional class (Schoenfeld, 1989). GSP puts geometry exploration tools directly in the hands of students, enabling them to test whether their geometric constructions work in general or whether they have discovered a special case of the original construction. This software also has the capability to link synthetic constructions to analytic equations, and co-ordinate representations. Furthermore, challenging and time consuming mathematical problems could be easier through dynamic software (Lappan and Winter, 1984). As a result, GSP is used for exploration and guided or open-ended discovery—enabling students to test their conjectures and be more engaged in their learning.

When the literature was searched**,** on the use of Geometer's Sketchpad, the studies investigated the geometric learning of secondary school students during instruction, on the basis of the Van Hiele model, with GSP as a tool (Battista, 2002; Choi-Koh, 1999). Furthermore, these studies were conducted with small groups (case studies) and investigated how students moved to higher levels of Van Hiele geometric thinking. Battista (2002) studied with three children in learning of quadrilaterals (parallelograms), and Choi-Koh (1999) studied with one child on learning of types of triangles (scalene, equilateral and isosceles). Shaw and Durden (1998) used case study method to investigate a cerebral palsy student' geometry learning, especially how she understands angles under the usage of GSP. Only Dixon (1997) used GSP in the computer lab environment in the learning of the concepts reflection and rotation. When we looked at these case studies, the topics of polygons especially the types of quadrilaterals and triangles were investigated (Battista, 2002; Choi-Koh, 1999).

Having established these facts, it seems logical to examine the effect of the dynamic instructional environment (based on the use of GSP) on students' understanding of and performance in lines, angles and polygons (triangles, square, rectangle, parallelogram), compared to traditional learning environment. Furthermore, this study, adding to other relevant studies, investigated the effects on retention level. The theoretical framework of the dynamic instructional environment was based on an interactive drawing environment in which the students can construct and explore the geometric figures by manipulating them. This ability of manipulating a figure and observing the effects on measurements enable students to discover relationships for the development of conjectures for themselves and to make generalisations.

The following research questions were addressed in this study: (1) Is there a significant mean difference in the performance scores of the EG and the CG students on geometry prior to the treatment on geometry? (2) Is there a significant mean difference in the performance scores of the EG and the CG students on geometry upon the completion of the treatment? (3) Is there a significant mean difference on the gain scores, calculated by using post and delay performance scores, of the EG and the CG students? 4) Is there a significant mean difference on the retention level of the EG and CG students? 5) How rich is the concept image of the two groups, as expressed in the justifications produced? In particular, what is the status of the prototypical examples?

The concept image is the total cognitive structure that is associated with the concept, which includes visual representations, impressions, experiences, and all the mental pictures associated with the concept name (Tall and Vinner, 1981). During the mental processes of recalling and manipulating a concept, some special examples, particularly figures in the case of geometry, are brought into play, consciously and unconsciously affecting the meaning and usage. These special examples often called prototypes. The prototype is a result of our visual-perceptual limitations that affect the identification ability of individuals, and individuals use the prototypical example as a model in their judgements of other instances (Hershkowitz, 1989, 1990; Shwarz and Hershkowitz, 1999). In the case of geometry, students sometimes derive a concept from its prototypical examples and hence it has relevant (critical) attributes and non-critical attributes (those attributes that only some of the concept examples possess).

Method and Procedure

Sample

The participants in this study were 63 7th grade students (32 girls and 31 boys) in a state elementary school. There were two 7th grade classes in the school, which were being thought by the same mathematics teacher. One group constituted the experimental group (EG) and the other the control group (CG). The groups were selected by randomly. EG consisted of 31 and CG consisted of 32 students. Students' ages in both group ranged from 12 to 14. The EG students were composed of 15 girls and 16 boys, whereas the CG students were composed of 17 girls and 15 boys.

Instrument

The geometry performance test (GPT) was prepared to investigate 7th grade students' performance on geometry. GPT includes twenty-two questions, some of which having some sub-tasks. In the GPT, questions 12, 15, 19, 20, 21 and 22 were taken from the Van Hiele Geometry Test developed for Cognitive Development and Achievement in Secondary School Geometry Project (Usiskin, 1982) and the researchers prepared the rests. The test was based on geometry topics given in the 7th grade: Lines and Planes; Angles and Types of Angles, and Polygons (Triangles and Types of Triangles, Parallelogram, Rhombus, Square, and Rectangular).

Each task in GPT was analyzed by giving 1 for each correct answer and 0 for each incorrect answer. In addition, explanation required under each question was also taken as a different task, and therefore scored as one or zero. The maximum score for test was 82.

The test including 82 tasks altogether was administered to the subjects as a pre-test, post-test, and delayed post-test, allowing 50 minutes each time. Post-GPT results yielded a Split-Half reliability coefficient of internal consistency of 0.74.

Procedure

The study was conducted in a course designed to teach the regular topics of geometry that are normally covered in 7th grade. The treatment in the dynamic instructional environment included exploring and manipulating geometric concepts (line, angle, and polygon) based on productive three-way interaction between teacher, students, and computer through subsequent activities, named as Sketchsheets. Students worked on these activities at computers provided at the computer-lab. The Sketchsheets were prepared by the researchers to permit student inquiry, while guiding, prompting, and helping them to identify relationships and make conjectures. The traditional instructional environment was based on a text-book based approach using chapters related to the lines, angles, and polygons from *İlköğretim Matematik 7* (Yıldırım, 2001), the adoptive text-book for the 7th grade students in the study. The students in both groups were taught the same mathematical content at the same pace in the second term of the 2001-2002 academic year, lasting five weeks. There were four mathematics classes in each week, two hours in a day, lasting 40 minutes each.

The students in the EG and the teacher were taught to use the Geometer's Sketchpad prior to the treatment. All training was conducted by the first researcher and lasted approximately two class hours. During training, the students were required to do hands-on activities to aid constructing points, lines, angles, and polygons on the computer. For the study, some explanations related to their native language were added to GSP in order to prevent language being a problem and to help students to use GSP effectively.

The Geometry Performance Test (GPT) used in this study was developed by taking into account the findings of previous studies. The GPT was administered to both groups of students as a pre-test, post-test, and delayed post-test. The pre-GPT was administered to the students prior to the treatment to ensure that two groups were equal in understanding of lines, angles, and polygons at the 0.05 level of significance. The post-GPT was administered upon the completion of the treatment to determine the effects of the dynamic instructional environment on students' performance. Finally, delay post-GPT was given five months after the termination of the treatment to both groups in order to investigate the effectiveness of dynamic instructional environment on students' retention level.

GPT was piloted, in the first semester of 2001-2002 academic year, on three 8th grade students by using face to face interview, identified by their mathematics teacher as having 'above average ability', 'average ability' and 'below average ability' in mathematics according to their grades in mathematics. The purpose of this pilot study was to examine students' difficulties on understanding the questions and identify misconceptions, and according to these results, to prepare Sketchsheets for the main study.

The Statistical Package for Science (SPSS) was used to conduct statistical procedures on the data. After scoring each task on pre, post, and post-delay GPT, frequencies of each task according to the scoring criteria were computed. Furthermore, descriptive statistics were calculated for each test. To calculate group differences on pre, post and delay-GPT, independent samples t-tests were conducted.

Both of the classrooms and computer sessions were observed and recorded with camera.

Treatments for the Experimental and Control Group

Treatment for the Experimental Group. Prior conducting the treatment, to familiarize the EG students and the teacher with GSP and its proper usage, a couple hours of hands-on instruction and practice were given at the computer lab as mentioned above. At the end of this practice all students were capable of constructing points, lines, angles and polygons on the computer. This practice, however, was not adequate for the teacher. As she mentioned herself, she was not experienced in using computers. Not causing this to be a problem, the first researcher helped the students in the computer lab under the supervision of the classroom teacher during the whole treatment. Following the three class hours at the computer lab, the students and the class teacher discussed together the findings come out in the computer-lab and the teacher made a brief introduction to a new topic in the regular classroom in one class hour. The lab contained 18 computers located in the U shape and so students worked in pairs at the computer.

18 Sketchsheets to be used together with the GSP were developed for this study. The majority of the Sketchsheets were of an investigative nature. Investigations guided students toward discovering a specific property or small set of properties. Students developed personally meaningful geometric concepts by exploring, manipulating and transforming the geometric shapes. For instance, Sketchsheet 12 (Figure 1), aimed students first, to select equilateral triangle from the given triangles on GSP screen, then to find the features of angles, sides and the relationship between angle and side of equilateral triangles and finally to discover that these findings are acceptable for all equilateral triangles by manipulating them.

At each computer session daily sketchsheets were distributed to the students. Upon a completion on working on each sheet, students wrote their findings on their Sketchsheets. Then, students were asked what their findings and conclusions about that activity and then the whole findings were written on the board. In this way, the students discussed and interpreted the findings. With this discussion, one sketchsheet was completed and the following Sketchsheet was distributed to the students. The results of the completed sketcsheets also discussed in their regular classroom and homework was assigned to the students from the textbook " İlköğretim Matematik 7" written by Yıldırım, (2001). All activities in the Sketchsheets were completed in this procedure.

Treatment for the Control Group. The method used in this class was traditional method. In general, the teacher explained the concepts by writing them on the board, and then allowed students to write them on their notebooks. While starting the lesson, she always reviewed the previous lesson by writing the important rules or procedures. Then, the lesson was continued either by asking to the students to do some similar exercises, which were worked in previous lesson, or writing a new rule or a new definition. Students in the CG were taught using chapters from their textbook (Yıldırım, 2001). The teacher usually used ruler for drawing lines and protractor for measuring the angles. While she was drawing lines or angles, students also drew them on their notebooks by using the same tools. At some exercises, one student among the volunteer students was called to come to the board and show his/her solution of the exercise. Subsequently, the teacher explained again the solution of the exercise upon the completion of the solution by the student. The teacher assigned home works from the textbook each time when the topic was completed.

Results

Descriptive statistics are given in Table 1. Independent samples t-test was carried out in order to examine whether there was a significant mean difference between the EG and the CG students' pre-GPT scores. The results showed that there was no significant mean difference between the EG and the CG with respect to the pre-test prior to the instruction of geometry ($t=-0.168$, $p=0.867>0.05$). This result also confirmed by the Effects Size

(ES= 0.04< 0.5), as the ES less than 0.5 shows not a significant mean difference. That is, both groups were equivalent in geometrical performance at the commencement of the experiment.

Subject: Types of Triangles

Sketchpad Activity 12

1) Select equilateral triangle from your computer screen.
2) Measure the interior angles of this triangle.
3) What kind of relationship between these angles?
4) Are these angles acute or obtuse?
5) Measure the length of sides of this triangle.
6) What kind of relationship between three lenght of sides?
7) How the measure of angles are changing while you change the length of sides by dragging the triangle?

Result: Please define Equilateral Triangle according to your findings.

Figure 1: Sketchsheet 12

Table 1: Descriptive Statistics for pre-GPT, post-GPT and delay-GPT scores for the EG and the CG

	Pre-GPT		Post-GPT		Delay-GPT	
	EG	**CG**	**EG**	**CG**	**EG**	**CG**
N	31	32	31	32	31	30
Mean	42.9	43.25	59.8	48.4	55.06	47.4
Std. Deviation	9.25	7.07	8.41	6.27	11.08	9.3

Independent samples t-test was carried out in order to examine whether there was a significant mean difference between the EG and the CG students' post- GPT scores upon the completion of the treatment on geometry. The results showed that there was a significant mean difference between the EG and the CG scores with respect to the post-test ($t=6.15$, $p=0.0<0.05$, $ES=1.55>0.5$). This means that the EG students showed a test mean score that was significantly higher than the CG students.

Finally, the difference scores between post-GPT and delay-GPT were calculated in order to examine whether there was a significant mean difference on the gain scores between the EG and the CG. When t-test was conducted with these scores, it showed that there was no significant mean difference between the EG and the CG ($t=1.305$, $p= 0.197>0.05$). But when the independent samples t-test was carried out with delay-GPT scores, it was found that there was a significant mean difference between the EG and the CG ($t= 2.92$, $p = 0.005 < 0.05$). The magnitude of the Effect Size ($ES=0.75>0.5$) also confirms this result. Furthermore, most of the EG students kept their true definitions in the delay-GPT. We can conclude from these results of the delay-GPT, dynamic computer instruction also raised scores on follow-up examination given several months after the completion of the instruction, but these effects were not as high as the immediate effects of dynamic computer instruction.

In spite of the rising in the EG scores, the CG scores did not rise in the same degree of the EG or they could not protect the exist percentages.

Discussion and Conclusion

A comparison of the pre-and post-test means of the students indicates that the treatment resulted in marked improvement in their performance in lines, angles, and polygons in the EG. Similarly, the significant differences in the results of the post-GPT indicated that overall the students in the EG performed better than the students in the CG**.** Also when the two classes of students' answers and their written explanations in GPT were deeply analyzed, it is seen that the students acquired conceptualisation of lines, angles, and polygons better in the EG than in the CG. This result revealed that the dynamic instructional environment has facilitated to the students' better understanding of the geometric concepts thought. Working in this environment helped students build increasingly sophisticated mental models for thinking about geometric shapes. Such work also encouraged and supported students' development and understanding of the property-based conceptual system used in geometry to analyse shapes. It encouraged students to move to higher levels of geometric thinking instead of having to memorise a laundry list of shape properties. The dynamic instructional environment involved students as conceptualising participants, not massive spectators in the process of doing geometry. These findings support the findings of previous studies (Battista, 2002; Chazan, 1988; Choi-Koh, 1999; Dixon, 1997; Kakihana and Shimizu, 1994; Yusuf, 1991).

Adding to these results, analyses of the students' explanations given in some of the questions indicated that EG students' understanding was deeper in content than CG students. One of the most important results of this study is that students instructed with the dynamic geometry environment, got rid of the prototype phenomena. From the EG students' written responses in pre and post tests, it was seen that there is a crucial difference in students' definitions and explanations. Although the definitions given in the pre-test included non-critical attributes of shapes, most of the EG students' definitions in the post-test included critical attributes not non-critical attributes of the concepts. This difference base on the feature of visualisation of GSP especially the distinction between a drawing and a constructing. For example, in a classroom, when a teacher draws a figure on the board and declares to students the figure is a parallelogram ABCD, the teacher is trying to say the students " let ABCD represent a parallelogram, and let all the properties inherent in a parallelogram be attributed to figure ABCD".

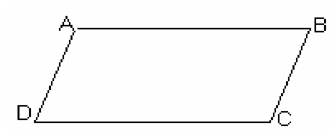

Figure 2: A figure which could represent a parallelogram

The figure that presented in Figure 2 is the collective drawing of a parallelogram in geometry lessons. The students expect to understand this parallelogram is a generic parallelogram, and will remain a parallelogram no matter what its orientation or scale. When the figure drawn while introducing parallelogram, *parallel opposite sides not having right angles*, has been found to be important in conceptual judgement. Most of the students in pre-GPT did not accept right angled shapes (e.g. rectangle, square) as a parallelogram. This generic shape of parallelogram causes students to have prototypical shapes on their minds. But, GSP allowed students to play with figure, and build dynamic and flexible geometric models by dragging the shapes. Also they created, explored, manipulated the parallelogram and transformed this parallelogram to a rectangle or a square. By this way, they have discovered a special cases (e.g. rectangle, square) of the original construction of a parallelogram Furthermore, students made their own judgements and acquired their own conclusions, and became distant from the prototype phenomena. In contrast, when students create a parallelogram using rulers and protractor as in the control group, students could not immediately create a parallelogram having all the correct features and reshape, resize, rotate, translate, and measure a desired characteristic of the figure at each stage of its transformation. Similarly, when students create a parallelogram with a manipulative and were asked to reshape, resize, rotate, translate, and measure a desired characteristic of the figure at each stage of its transformation, students might have difficulty to generalize their findings to all figures (Perham et al., 1997) and it may take quite a lot of time to complete particularly the tasks which require variation. Dynamic media, however, make variations easier to achieve (Kaput, 1992).

Like this example, students had prototypical shapes on their mind. But, during the dynamic instructional environment with GSP, students experienced every orientation of the shapes by dragging. This "drag" feature allows users to modify the objects of the constructions. By this way, the students starting from the visual considerations of the geometrical shapes, had, towards end of the process, developed connections between the figures and their properties, and formed hierarchical relationships between different classes of shapes. This important result of the study supports that of Clements and Battista (1992), who reported that geometry software packages had a better knowledge of geometric concepts, and a richer understanding of conjecturing skills.

Delay Post Test results showed that the dynamic instructional environment has a significant effect on students' retention level. The dynamic instructional environment raised scores on follow-up examination given several months upon the completion of instruction, but the retention effects were not as clear as the immediate effects of the dynamic instructional environment. This is consistent with findings from earlier studies (e.g., Kulik, Bangert, and Williams, 1983).

From the data and it's analysis, the conclusion drawn that seventh-grade students in the EG had a higher performance level in learning geometric concepts using GSP, and a better retention level.

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