# Mathematical Modelling: a path to political reflection in the mathematics class

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## Abstract

This paper describes the construction of pedagogical environments in mathematics classes, centred on mathematical modelling and denominated "investigative scenarios", that stimulate students to investigation, to formulation of problems and to political reflection, as well as the sharing of acquired knowledge with other persons of the community. The paper is based on the application of modelling as a teaching and learning strategy and on the pedagogical work with teenagers that were in an assisted freedom program. Both were accomplished in a scenario built with ten volunteer students taking Calculus in the Computer Engineering course, in the year 2003. Among the main results we emphasise the academic maturing process for the student, how competent he gets in making models, accomplishing simulations, his perception of the relation between the mathematical learning and everyday situations and political reflection about the results from working with modelling as much as about his participation in the community work.

Keywords: mathematical modelling, investigative scenarios, political reflection.

## 1. Introduction

When the teacher uses in class pedagogical models developed exclusively in order to fit problems or exercises to the curricular content, it is possible to identify an alienating view, contrary to Paulo Freire's (1978) liberating thought. There is no dialogue, just explanation and a natural and spontaneous acceptance of the speech given by those who know, to the ones that do not.

In such models, education becomes only an act of depositing, in which the students receive and the teacher deposits. In this educational model, named by Paulo Freire (1978) as "bank education", there is no space for discussion, reflection and questions about the social and political meanings of this learning process that can contribute to the student's critical background. In contrast to this "bank education", the author proposes an education that searches for challenges, stimulates the formulation of problems, investigations, discoveries, revelations, and instead of inhibiting criticism, encourages it.

All of Paulo Freire's considerations (1978; 1982; 2003), along with Ole Skovsmose's studies on critical mathematical education (1996; 2001), led us to the thought about pedagogical environments in mathematics classes that, not only allow the learning of the curricular content, but also encourage students to question and to discuss this content, to investigations, to political reflection on the results acquired

from these investigations and discussions, and still to the student's participation in activities outside the classroom, in a way that all the acquired mathematical knowledge could be shared with other sectors in the community. We call these environments "investigative scenarios" and, following Skovsmose (2000) and Barbosa (2001), organise our scenarios supported by mathematical modelling.

We see such investigative scenarios as alternative spaces built in the classroom, formed by volunteer students who are willing to participate as a result of their belief in the pedagogical proposal and its work dynamic. In them, we focus our attention on both the student's academic growing up process and his political and social background. The use of politics in this study does not refer to questions about elections, ideologies or doctrines, but, in agreement with Mellin-Olsen (1987), the term politics is related "to the position that human beings act, participate and survive in their world as political human beings" (p. 38).

Analysing the possibilities of mathematical modelling in the context of investigative scenarios as teaching and learning strategy and political experience is the main objective of this study.

In this paper we highlight the investigative scenario "Mathematics and Citizenship", built by the first researcher in a Calculus discipline, in the year 2003, with ten university students from the Computer Engineering course at Puc-Campinas, in Brazil. These students, aged around 18 and 19 years-old, volunteers, have middle class families, went to good schools and had an appropriate school background, especially in mathematics.

The theme of the scenario, related to taxation and income tax, was proposed by the researcher due to its importance to Brazilian society at the time of the study. The activities in the scenario were divided in two stages. In the first, we focused on working with modelling (investigating the theme, studying the "income tax" model and its relation with the curricular content, formulating new models, discussing the results). In the second, we planned to highlight the political component of the scenario that began when modelling was applied and followed the pedagogical work based on the applications of mathematics in everyday life, developed by the students with teenagers in an assisted freedom program from the city of Campinas.

The research based on working with projects centred on investigation, reflection and criticism, whose subjects are the students in their classroom, requires constant interaction between the researcher and his students and, due to this involvement, we direct this qualitative study to the action-research field.

We begin this article by focusing on mathematical modelling as a pedagogical strategy in the context of investigative scenarios. Then we plan to describe and analyse all the work accomplished in the scenario "Mathematics and Citizenship".

## 2. Mathematical modelling in the context of investigative scenarios

According to Pierre Lévy's studies, related to distance learning, "educational systems find themselves today constrained by the quantity, diversity and speed of the evolution of knowledge" (2000, p. 169). In his analysis, Lévy adds that strict and uniform courses that are not related to the students' real need tend to become less interesting and even intolerable to most of them.

However, under the principles of critical education, these courses, regardless of their whole organisation, distance (with or without the Internet support) or present,

tend to be equally unproductive if they are concerned essentially about knowledge transmission. Therefore working with projects is, on one hand, breaking away from just obtaining knowledge. And, on the other hand, takes a pedagogical view, which is directed to the fundamentals of critical education and accomplishment of a learning based on active participation of the students from the study of problem-situations in their everyday.

The working with projects (Skovsmose, 2001), projects pedagogy (Hernández and Ventura, 1998, Boutinet, 2002), environment study, working projects and themes work (Hernández, 1998), project-oriented approach (Mellin-Olsen, 1987) are terms used to describe a pedagogical way of work in which education is a life project, and not only a pragmatic preparation to the future through the transmission of culture and knowledge. In this conception, a study program is developed from the curricular organisation and development, clearly intending to transform the student, from object to subject. In working with projects, it can be noticed that the formulation of problems and the need for investigation give value to discussion, reflection and criticism.

In Mathematical Education, this investigative work based on project activities is often accomplished through the pedagogical application of Modelling. According to Bassanezi (2002), one of the first people to introduce modelling in Brazil and also one of the most important researchers on the theme, "modelling is a dynamic process used to validate the mathematical models and consists, essentially, in the art of transforming real problems and solving them, interpreting their solutions in the real world language" (p. 24). Still according to Bassanezi, "mathematical modelling can be used as a process to the solution of the most varied range of problems related to Applied Mathematics or as a strategy of teaching-learning" (p. 32).

Galbraith (1995), in the proceedings of the ICTMA-6[1], classifies the work with modelling in the classroom in three stages: general applications, structured modelling and open modelling. In each of these stages, the author talks about different pedagogical works with modelling. Having included general applications in the work with models, the author considers standard applications found in conventional texts and after mathematical discussion needed to obtain solutions, the original aspects of modelling are introduced (meanings, limits, discussion about the mathematical results, decisions, generalisations, etc.).

In structured modelling, real questions, that is, questions related to reality are considered alongside the application's formulation. In this stage, intermediary to open modelling, there is strong assistance from the teacher, supporting the model's construction and its solution as well as in guidance to the search for information.

In open modelling, the students are encouraged to work with real situations that involve them in a certain way, searching for information, developing skills of formulating models from complex realities, finding answers to these models, interpreting and adapting these answers and promoting discussion about the results. It is in this stage that the modelling process in the classroom, in its most complete form, occurs. To the author, one stage does not exclude the other, being in fact an incentive to their integration.

When modelling is proposed as a teaching and learning strategy, most researchers, following Bassanezi and Galbraith, give emphasis to the construction of

[1] ICTMA-6 – International Conference on the Teaching of Mathematical Modelling and Applications, held at the University of Delaware, USA, August, 1993.

models and to applications of the mathematics in everyday life. From these two methodological actions and based on the principles from Critical Mathematical Education, we planned to explore other possibilities in the pedagogical work with modelling and found in the "landscapes to investigation", proposed by Skovsmose (2000), a way to work with them.

The "landscapes to investigation" are, according to the author, environments built in the classroom, centred in modelling or in projects, in which students are invited to propose questions, search for explanations and think about the results achieved[2].

As stated by Skovsmose (1988; 2001), reflection in Mathematical Education is related to reflective knowledge (ability to think about the use of mathematics and evaluate it). He writes, "it is related to evaluation of the consequences from technological enterprise (the capacity to apply mathematics and competencies in the modelling construction)" (2001, p. 116).

As we see, Skovsmose's considerations about reflection (or reflective knowledge) are deeply concerned with the student's preparation for thinking about technological knowledge in Mathematical Education. We consider his preoccupations absolutely legitimate, and also share them. However, when we characterise the whole investigation scenario, we amplify the concept of reflection and also include reflection that come from investigative work, including the participant's growing up process, discussion among them (mathematical or not), their changes in thought, the way they act and their involvement as citizens. These are all a consequence of this growing up process and discussion.

We are interested, in this characterisation, in reflection that come from applications in some context (social, political, economical, educational, the classroom, the school) that have some relation with the actors involved. We believe, as Freire (2003), that education aiming to develop social and political responsibility in students is one of the main tasks of a critical education.

Hence, we use Skovsmose's thought in order to propose investigative scenarios with this political component into the classification presented by Galbraith, not as an intermediary stage, but as a working environment whose main instrument to pedagogical action is modelling. We see a investigative scenario as a pedagogical environment strongly related to critical learning, in which knowledge based on investigation, inquiry and reflection about what is being learned, how it is learned and what it is learnt for, is actually given the same importance as the learning of skills and concepts.

## 3. The investigative scenario "Mathematics and Citizenship"

The discussion on the theme Taxation and Income Tax, appropriate at the time of the study and of great interest to Brazilian society[3] and the reason for the students' involvement in pedagogical and community work were responsible for the title "Mathematics and Citizenship", given to the scenario. Our interpretations and results

[2] Skovsmose proposes landscapes to investigation in contrast to the model based on exercises, which he calls "exercise paradigm" and which D'Ambrosio (2001) calls "formal education".

[3] In 2003, two major reforms were established in Brazil and attracted great attention from the media and in discussions that took place in political scenarios. These are social-security and income tax contributions.

obtained after the scenario's implementation, and that go along with its description are based on data proceeding from our observations, interviews with the participants of the investigative environment, and reports presented by the students.

### 3.1. The Project "Taxation and Income Tax"

After the group was formed and there was good agreement on the work's objectives (theme, activities, timetable, procedures, etc.), we started thinking about the traditional aspects in modelling. These are a search for the information required for the understanding of subject investigated, the construction of mathematical models, the relation between these models and the curricular content and reflection on the results obtained. In this phase we alternated pedagogical work along with Galbraith's stages.

We consider: -

- distributing tasks,
- organising the timetable,
- constructing a webpage to publicise the results (http://docentes.puc-campinas.edu.br/ceatec/otavio/matci/),
- investigating the taxation in Brazil (its evolution throughout the years and discussion about changes in the Brazilian system followed by the consequences of this reform to society),
- writing reports and presenting the results (oral to the classmates and via website to the rest) as the students' first steps towards their academic growth.

Alongside the investigation about taxation, the students began to study the income tax model. The first step was to understand its format, and the information was obtained from the Federal Reserve website. In the Brazilian system, annual incomes lower than R$ 12.696,00 are tax-free, the group between R$ 12.696,01 and R$ 25.380,00 is taxed at 15% and the ones that are above R$ 12.696,00, at 27,5%. Amounts of R$ 1.904,40 and R$ 5.076,90 are deducted from the sum calculated in the $2^{nd}$ and $3^{rd}$ of these groups.

After the problem was identified, but before "mathematizing" the income tax calculation, the students formulated some questions to guide them while developing the work, such as: why is the system composed of groups and not a single percentage? Why are there only three groups and what is the reason for percentages of 15% and 27,5%? What is the meaning of the amounts of R$ 1.904,40 and R$ 5.076,90? Which mathematical model represents the calculation of the income tax? What other models can be built? Which comparisons can be made?

In order to help them understand the system, while testing new models and performing simulations, the members gathered information about annual incomes from their families and friends.

The choice of the function defined by several mathematical sentences as a model of the income tax can be explained by the existence of groups in this taxation system. And the linearity of each mathematical sentence derives from the application of a percentage on the income, followed by subtraction of a fixed value. The income tax as a function of annual income x, resultant from this study is shown in figure 1.

$$IR = f(x) = \begin{cases} 0 & if \quad x \leq 12.696,00 \\ 0,15x - 1.904,40 & if \quad 12.696,00 < x \leq 25.380,00 \\ 0,275x - 5.076,90 & if \quad x > 25.380,00 \end{cases}$$

Figure 1: Mathematical function to calculate income tax in Brazil

The process of constructing this model was followed by intra mathematical discussion. This was especially related to the course's program and dealt with linear functions and those defined by several mathematical sentences, the importance of function graphics as an instrument to visualisation and interpretation, the study of a continuous function, variation tax and derivative. We highlight the equality between the lateral limits surrounding the border points of each group (R$ 12.696,00 and R$ 25.380,00) and, therefore, the continuity of the function at these points as a condition to obtain the payments deductible after the percentage is applied (R$ 1.904,40 and R$ 5.076,90).

After the mathematical results were achieved, we moved on to discussions related to the application of model 1 (the present model) and, consequently, the proposal of others with even more groups and percentages: model 2 (system with six groups and a variation of 5% in steps of 5%), model 3 (system with four groups and a variation of 15% in steps of 15%) and model 4 (introduction of a new group with percentage of 35% to the actual system[4]). At the same time, from the many simulations performed with the help from Excel, comparisons among all these models were accomplished. The table in figure 2 (values in Reals[5]) shows a clipping from these simulations and is followed by the differences with the current Brazilian system.

| Incomes | Income Tax | | | | Differences | | |
|---|---|---|---|---|---|---|---|
| | Model 1 | Model 2 | Model 3 | Model 4 | (2) and (1) | (3) and (1) | (4) and (1) |
| 15.000,00 | 345,60 | 450,00 | 345,60 | 345,60 | 104,40 | 0,00 | 0,00 |
| 25.000,00 | 1.845,60 | 2.000,00 | 1.845,60 | 1.845,60 | 154,40 | 0,00 | 0,00 |
| 35.000,00 | 4.548,10 | 4.000,00 | 4.788,60 | 4.548,10 | -548,10 | 240,50 | 0,00 |
| 60.000,00 | 11.423,10 | 10.800,00 | 13.969,60 | 12.116,10 | -623,10 | 2.545,5 | 693,00 |
| 75.000,00 | 15.548,10 | 16.200,00 | 20.719,60 | 17.366,10 | 651,9 | 5171,50 | 1.818,00 |
| 100.000,00 | 22.423,10 | 27.200,00 | 31.969,60 | 26116,10 | 4.776,90 | 9546,50 | 3.693,00 |
| 125.000,00 | 29.298,10 | 40.800,00 | 46.446,60 | 34.866,10 | 11.501,90 | 17.148,50 | 5.568,00 |

Figure 2: Simulations related to the calculation of income tax in four situations

The results of these simulations and the study of models adopted in other countries (which stimulated the construction of new systems and, therefore, the accomplishment of new simulations), led the students to think about tax unfairness in the Brazilian system. The features of concern are - the existence of only a few collection groups, and the linearity of the last formula, which caused the same

[4] Including a 35% percentage in the present model is one of the alternatives studied by the government

[5] Real is the Brazilian currency (1U$ = R$ 2,90).

percentage collection from all incomes greater than R$ 25.380,00. The linearity, suitable for many models, is unsuitable for financial matters related to this kind of tax.

If this present model were to be maintained, it would make sense, as proposed by one of the students, to think about another function, perhaps a exponential. However, in consideration of all the simulations performed and the study of models adopted in other countries, the group decided it would be more interesting to keep the present income tax formula, though increased by one or two new groups with higher percentages. The simulations showed that the introduction of a new percentage group of 35% would produce a reduction in contributary unfairness.

Talks given by experts of the Brazilian institute responsible for taxation, organised and conducted by the work group, contributed directly to the debate on specific aspects related to the theme and also led to new reflections. We highlight the following: -

- the link among taxes and citizenship (democracy, rights and duties, both for citizens and State),
- matters of concealment and corruption and the harm they cause to society,
- unfair income distribution, responsible for the low number of tax-payers (nowadays, according to these specialists, about 80% of declarers are tax-free because they are included in the first group of the income tax function, 12% of them are taxed at15% and the other 8%, is taxed at 27,5%) and
- the importance of the guarantee of a minimum income (or inverse tax) to all citizens as an instrument to even inequality.

Besides competence in the construction of models and applying mathematics, when the students debated such points, they showed they were prepared to be citizens, that they thought about mathematics as an instrument to create and solve problems and that they see it as a tool to analyse critical aspects of social importance. According to Skovsmose (1996), this kind of reflection can be found in Critical Mathematical Education.

As the project was not incorporated in the formal process of teaching Calculus, its weight in the students' course evaluation was too small. Besides, topics such as continuous functions and derivatives, directly related to the modelling activities, were developed in normal classroom teaching and, consequently, the mathematics required to build the models "was available" for the students in the project. For these reasons, we were not worried about establishing parameters to evaluate the modelling work developed by the students.

However, when working with modelling, we see the applications of mathematical concepts, their relation with the theme, the construction of models and the simulations with support from computers as important contributions to the perception of the relation between mathematics and reality. At the moment the student notices it, he or she breaks the (pre)-conception that mathematics is purely abstract and discovers the presence of its practical side in everyday life. He or she finds meaning to the learning and strengthens their mathematical knowledge. This perception is also reported by D'Ambrosio (1991), to whom mathematics with a meaning is a live mathematics, that keeps growing with the student while he himself is developing his ways of working with reality.

Then, we observed that the project members found meaning for their learning about limits and continuous functions when they realised the importance of a function

defined by mathematical sentences as a mathematical model for the calculation of income tax. They also saw the need for equality between lateral limits in the border points of each group in order to prevent distortions in taxation.

In the same way, students found meaning for learning the derivative (as a function and as rate of change) and for the relation between derivative and continuity. Also they learnt that, while every differentiable function is continuous, not every continuous function is differentiable. This became clear when they associated the step function graph of the derivative of the income tax function (composed by percentages charged for each income group) with the changes in taxation that come from the passage from one group to the other. They also learnt derivatives do not exist at the border points.

### 3.2. Pedagogical work with teenagers from COMEC

The discussions student had relating the modelling of social policy and income tax contributions to their learning of mathematics led to the introduction of the political-reflective components in the investigative scenario. Through the community pedagogical work carried out by the scenario members with teenagers from the Campinas's Centre of Younger Orientation (COMEC), we can clearly see this component. Because these teenagers committed some infraction of the law, they were required to attend a program of assisted freedom. In this program there are educational activities such as professional training courses or student support, aiming to reform the youngsters via citizenship, self-esteem and preparation for employment.

The teenagers from this program, in contrast to the ones from the Computer Engineering course, had flaws in their school background, mainly concerning mathematics. They also came from low-income families. When organising the scenarios, we chose to highlight these differences, because, like Skovsmose (2001), we believe that to be critical, education ought to fight social contradictions.

In this community project, the scenario members received in the University's Laboratory of Mathematical Teaching, four teenagers (three boys and a girl) from the program and developed for three weeks activities related to mathematical applications in everyday situations and basic knowledge about websites. While working with the Internet, in addition to theoretical concepts, use of e-mail (each member created their own e-mail) and of the Internet as a whole, the participants supported by members from the scenario, built a website to show and publicise their work. This site can be accessed through the icon "Mathematics at school" ("Matemática na escola") in the scenario's homepage "Mathematics and Citizenship" (Matemática e Cidadania).

Activities with mathematical applications, organised by the researcher responsible for the project, were created from real problems, extracted mostly from the books from Imenes & Lelis (1997). Such problems, divided in five groups and arranged by increasing difficulty, involved working with fractions, percentages, statistical topics, functions and varied graphics. Since the group was quite heterogeneous when it came to mathematical knowledge, we chose individual work with the support from the scenario members.

To these teenagers, whose school models in mathematics classes were similar to the exercise paradigm (Skovsmose, 2000), attendance at this project offered for the first time the opportunity to experience practical and useful mathematics, with

problems that affected their everyday life. It is interesting to point out that the same situation happened also to most instructors-students.

For the students from COMEC, being in the scenario was part of a recovery and social participation program involving activities related to the Internet and mathematical application. For the students from Computer Engineering, it was all about their wish to obtain meaning for their learning and, through mathematics, to help teenagers similar in age, but with absolutely different lives and opportunities. To these students, spending time in volunteer work created an idea of being helpful to society and even helping someone who did not like mathematics and who, as a consequence of this, did not understand its concepts nor its algorithms. Several times we noticed smiles and satisfaction from the engineering students after they managed to explain a problem.

According to Machado (2000), among all the functions of Education, the one that seems most adequate is the one that leads to formation of the citizen and construction of citizenship. We agree with Machado, and highlight the fact that this educational work helps create an idea of citizenship to the members of the scenario. We could perceive from our work that university students have considerable interest in joining volunteer activities in order to mix personal activities (related to their development) with social activities, but who do not know how to. The projects in mathematics class offers to students the opportunity to get out of school and interact with the community. As one of the members said, "We always felt like joining volunteer projects, but we did no know what to do... To me, this work had a huge importance because, besides other contributions, I could clearly be a good citizen, do something useful, and from what I can tell, help society".

Even thought the teenagers on the assisted freedom program were not the subject of this research, we think it is very important to emphasise that their participation in the scenario helped to create a consciousness of citizenship in themselves and in our students. As a result of this consciousness, soon after finishing the project, the teenagers agreed to participate in social work involving training physically disabled people on subjects related to Informatics.

We also highlight the closure ceremony, when, with the presence of social assistants from COMEC, the group members received their certificates. For the teenagers, their diploma stated the training, which would be important in creating opportunities for their release, and possibly for the job market. For the students from the University, their diploma certified the pedagogical, social and volunteer side of the activity. The speeches given by representatives from both groups gave a lot of importance to the project, and the presence of the mother of a teenager, who touchingly gave thanks for the attention and support given to her son, and confirmed how much the work was able to achieve socially.

## 4. Final Considerations

In this study we have discussed mathematical modelling in the context of investigative scenario. Without disregarding modelling as a teaching and learning strategy centred in the construction of models and mathematical applications, we also discussed the political-reflective possibilities offered by its work in the classroom.

Among the main results from the scenario built into a Calculus course, we emphasised: -

- the academic maturing process for the student,
- how he becomes competent in creating models and accomplishing simulations,
- his mathematical learning and his perception of the relation between this learning and the models constructed,
- political reflection on the results from working with modelling,
- his participation in the pedagogical, community and volunteer work with the teenagers from COMEC and its contribution to citizenship.

Altogether, we emphasise that at the moment the teacher opts for this political view in applying modelling, he or she should be prepared to face challenges that go beyond the everyday in a mathematics classroom and the information of the program. The teacher should also be aware that he or she is going to be away from the "safety" of a traditional classroom, which Borba and Penteado (2001) and Skovsmose (2001) call the "teacher's comfort zone", and go into a risk zone, marked by the need to take a political stand, by questioning and by constant involvement with the students and activities outside school.

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